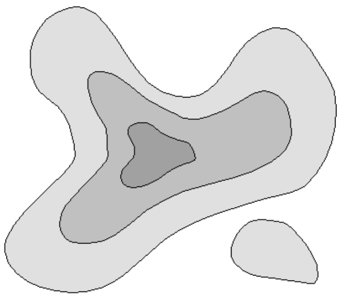
FOURPOT

Processing and analysis of potential field data using 2-D Fourier transform

User’s guide to version 1.3



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1. Introduction

FOURPOT is a computer program for frequency domain processing and analysis of two-dimensional (2-D) potential field data arising particularly from geophysical gravity and magnetic field measurements. The data can be regularly or irregularly sampled. In the latter case the data must be first interpolated on a regular grid.

The frequency domain operations include:

1. Frequency filtering (low-pass, high-pass & directed filtering)
2. Upward and downward continuation
3. Reduction to magnetic pole and equator (with rotation)
4. 1.st vertical and horizontal gradients (*d/dz*, *d/dx & d/dy*)
5. 2.nd degree gradients (d^2/dx^2, d^2/dy^2, d^2/dz^2, $d^2/dxdy$, $d^2/dxdz$ & $d^2/dydz$)
6. Rotational invariants (of gravity tensor) and “Falcon response”.
7. Horiz. gradient, Total gradient, Tilt gradient, Theta map & Max horiz. gradient
8. Pseudo-magnetic field and pseudo-gravimetric field
9. Gravity potential and magnetic potential
10. Hilbert transform (1D or 2D)
11. Sun shading and generalized derivative operator (with rotation).

Considering a continuous function $f(x)$ with continuous first derivatives the Fourier transform $F(k_x)$ and the inverse Fourier transform can be written as (Blakely, 1995):

$$F(k_x) = \int_{-\infty}^{\infty} e^{-ik_x x} f(x)dx \qquad \text{and} \qquad f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik_x x} F(k_x)dk_x \tag{1}$$

Considering discrete 2-D data $f_{kl} = f(x_k,y_l)$ the governing equations become:

$$F_{nm} = \sum_{k=1}^{N}\sum_{l=1}^{M} e^{-2\pi i(\frac{nk}{N}+\frac{ml}{M})} f_{kl} \qquad \text{and} \qquad f_{kl} = \frac{1}{NM}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} e^{+2\pi i(\frac{nk}{N}+\frac{ml}{M})} F_{nm} \ , \tag{2}$$

where N and M are the number of data values in x and y directions. The transform is complex which means that it consists of both amplitude and phase spectrum. The discrete 2-D Fourier

transform is computed using fast Fourier transform (FFT) algorithm (Claerbout, 1976). More information about FFT algorithm can be found from Press et al (1988), for example.

The Fourier transform represents a sum of sine and cosine terms having different spatial frequencies or wave numbers (k_x and k_y) that are based on data sampling (d_x and d_y) in x and y directions. The highest spatial frequency is the so-called Nyquist frequency (e.g., $\max(k_x) = 0.5/d_x$). The lowest frequency is based on the data coverage (e.g., $\min(k_x) = 0.5/[\max(x)-\min(x)]$. Considering that the reciprocal of the spatial frequency represents wave length ($\lambda_x = 1/k_x$), zero frequency means infinite wave length, i.e., constant level of the data.

Because of the properties of the Fourier transform (symmetry, linearity, shift and derivative properties) several computational operations can be performed in Fourier transformed frequency (k_x, k_y) domain more efficiently than in the spatial (x, y) domain. For more detailed information about Fourier transform methods in potential field analysis, please, see Blakely (1995).

2. Installing FOURPOT

The FOURPOT distribution file contains both 32-bit and 64-bit versions that can be run on a PC under Windows XP/Vista/7 system and a graphics display with at least 1280×1024 pixels in size. FOURPOT uses dynamic memory allocation and reserves memory for several auxiliary data matrices derived from the interpolated data. Thus, memory requirements may become critical when working with very large data sets. Unlike the 32-bit version which is limited to 1 GB of continuous memory the 64-bit version can use all available system memory. In case of large arrays, however, the CPU becomes critical factor because computation of the 2D FFT of matrices larger than 4096 by 4096 takes lots of time. The program has simple graphical user interface (GUI) that is used to handle file input and output, to initiate the computational operations and to visualize the data and the results. The user interface and the data visualization are based on DISLIN graphics library.

The program requires either FOURPOT.EXE and DISDLL.DLL (the 32-bit executable file and dynamic link library for DISLIN) or FOURPOT64.EXE (the standalone executable file for 64-bit Windows).

The distribution file (FOURPOT*.ZIP) also contains a description file (_README.TXT), this user's manual (FOURPOT_MANU.PDF), and some example data files (*.DAT). To install the program simply decompress (Winzip or 7Zip) the distribution file and move the resulting FOURPOT folder to a preferred place (e.g., C:\TOOLS). To be able to start the program from a shortcut that locates in a different directory or from the desktop you should move or copy the DISDLL.DLL file into the WINDOWS\SYSTEM (or SYSTEM32) folder or someplace else defined in the system PATH statement (*Control Panel/System/Advanced/ Environment parameters*). If you replace the EXE files of a previous version, please, remember also to replace existing DISLL.DLL and FOURPOT.DIS files.

3. Menu items

3.1. File menu

The *File* menu contains following items:

Read data (dat)	read in (irregularly spaced) data in default data format
Read grid (asc)	read in (regularly spaced) data in ASC (ascii grid) format
Read matrix (mat)	read in (regularly spaced) data in matrix format
Save current data (dat)	save the data of current view in general xyz column format
Save current grid (asc)	save the data of current view in a ASC grid format
Save current matrix (mat)	save the data of current view in a matrix format
Save ridge data	save the discrete data points derived from ridge analysis
Save profile data	save the data interpolated along a user defined profile
Save model file	save the initial GRABODS/MAGBODS model file
Read BNA map	read an overlay map file in Atlas BNA file format
Read disp.	read new graph parameters from a *.DIS file
Save Graph as PS	save the current graph in Adobe's Postscript format
Save Graph as EPS	save the current graph in Adobe's Encapsulated PS format
Save Graph as PDF	save the current graph in Adobe's PDF format
Save Graph as WMF	save the current graph in Windows metafile format
Save Graph as GIF	save the current graph in GIF file format.

These menu items bring up the standard file selection dialog of the operating system that is used to provide the filename for I/O operations. All data files (DAT, ASC, MAT) are in text format (see chapter 5). DISLIN cannot be used for direct output to a printer, but the graphs can be saved into image files (PS, EPS, PDF, WMF, GIF) for printing. The graphs are saved as they appear on the screen in A4 size and landscape mode (see Appendix A). The preferred output format is PDF or PS, because their resolution is much better than that of GIF and WMF files. The PDF format is particularly useful because Adobe's Acrobat Reader can be used to take bitmap snapshots at desired resolution.

3.2. Edit menu

The *Edit* menu contains following items:

Inverse→data	set current results (of the inverse FFT) as new initial data.
Subtract inverse	subtract the inverse results from the original data.
Add inverse	add the inverse results to the original data.
Color scale	change the color scale (rainbow, gray scale, etc.).
Padding mode	change the method used to pad and taper data
Data type	define type of potential field data (gravity vs. magnetic).
Miscellaneous	sub-menu for some additional settings.
Low-pass filter	perform (GUI guided or manual) low-pass filtering.
High-pass filter	perform (GUI guided or manual) high-pass filtering.
Direction filter	perform (GUI guided or manual) direction filtering.

Inverse→data menu item makes the current (inverse) results the new (interpolated) input data for successive frequency domain operations. For example, tilt gradient can be computed after upward continuation or reduction to the pole without saving the UC or RTP results first in a data file. *Subtract inverse* item allows computing the residual field as the difference between the original (interpolated) data and the upward continued or low-pass filtered data. Similarly, *Add inverse* item can be used to add the current inverse results to the original data. Although this item is not very useful, it could be used to enhance high frequency content of the data, for example.

The items in *Color scale* submenu are quite self-explaining - they change the color scale of the image and contour maps. *Comparable colors* item imposes the same limit values on the color scales of the original and the processed data, and thus, makes their comparison easier.

Padding mode submenu is used to demonstrate different padding and tapering modes. Padding is an operation where artificial or null values are added around the original data area (see Appendix B). The purpose of padding is to extend the data (*N* and *M*) to an even power of two (e.g. 64, 128, 256, 512 etc.) as required by the FFT algorithm. Tapering is an operation where the "padded" values are made such that they prevent rapid amplitude changes at the borders of the data area. This means that the data and their derivative are made more or less

continuous across the original data area and padded margins. Together padding and tapering effectively remove so-called Gibbs phenomenon as well as ringing and other artifacts in the inverse transformed data. Padding will be discussed more in chapter 4.3.

FOURPOT does not care what kind of data it operates on. Strictly speaking, pole reduction and pseudo-gravity, for example, should be applied to magnetic data only. For teaching and testing purposes the *Data type* menu item allows defining the actual data type (undefined, magnetic or gravity). In practice, this has effect only on the information text next to the graph, the scaling of the radial amplitude spectrum (see chapter 4.9), and the proper selection of the output model files (GRABODS vs. MAGBODS).

The *Miscellaneous* submenu contains following items:

Zoom/Clip area	select (GUI directed or manual) a rectangular area from the data
Contour/Image	change between contour map and image (pixel) map modes
Show/Hide points	show/hide the original data points in original data view
Meters/Kilometers	swap dimension labels between meters and kilometers
Unit conversion	scale distances units with user defined values
Data conversion	scale data values with user defined value
Step number/distance	swap gridding between steps number or step distance

The minimum and maximum coordinate limits of the data matrix are based on the original data coverage. The *Zoom/Clip area* item can be used to redefine and round the coordinate limits and place the grid nodes “nicely” and/or to select a smaller area from large map. The operation can be made either manually giving the x and y coordinates of the borders or interactively with the mouse. In the former case the program shows an input dialog where the user gives the x and y limit coordinates of the area. In the latter case the program enters interactive editing mode and the user selects the corner points of the data area using the mouse as described below. After providing new limit values the program redraws the data map and the gridding (see chapter 4.2) of the data can be made using new data limits.

Note: All interactive editing operations, including the zoom/clip, are made in FOURPOT in following manner. When editing is initiated the mouse cursor becomes a cross-hair cursor above the graph area and most of the GUI widgets become inactive (so that the user cannot escape the editing mode accidentally). The user then defines the location of the point or points by pressing the *left mouse button* and exits the editing mode by pressing the *right mouse button*. Depending on the task the program shows instructions on the screen on the first time.

Also note that in case of *Zoom/Clip area* operation, the user can revert back to the original limit values by clicking the right mouse button without any left mouse button clicks (or by giving a blank line in the manual input dialog).

The information text next to the graph may show the dimensions incorrectly unless the *Meters*↔*Kilometers* menu item is utilized. Although knowledge about the spatial dimensions is usually not required, the pseudo-gravity and pseudo-magnetic field need to know the real dimensions for the amplitude of the transformed field to be more or less "truthful".

Unit conversion can be used, for example, to scale all distances from miles into kilometers or from meters to kilometers. Likewise, *Data conversion* can be used, for example, to scale the data from mgal/m into Eötvös (mgal/km). It can also be used to reverse the sign of the data (e.g. the negative gradient of the potential). Both in unit and data conversion, the program shows an input dialog where the user provides a single character for a mathematical operation (+ = addition, - = subtraction, * or x = multiplication and / (or :) = division) and the numerical value. For example, string "`* 100`" will multiply all data by one hundred. The operator has to be the first character on the given line. An empty line or zero value cancels the operation.

Step number/distance is a mode switch for grid sampling that changes the meaning of *Step x* and *Step y* text fields in the left control panel. The original data are considered to be irregularly sampled and must me gridded (and padded) before FFT. When *Step number* mode is active the user provides the number of grid nodes (*Nx*, *Ny*) in x and y directions (integer values). When *Step distance* mode is active the user provides the distance between two grid nodes (*dx*, *dy*) in x and y directions (floating point values).

The remaining items in the *Edit* menu are related to frequency filtering operations that can be performed either interactively with the mouse or manually by providing the numeric values via GUI. Low-pass and high-pass filtering are the two most typical operations. Considering 2-D FFT, low-pass filtering removes (nullifies) values outside certain radius from the origin of the (amplitude) spectrum (see Appendices C, E and F). On the contrary, high-pass filtering removes low frequency data around and near to the origin of the spectrum. Combined use of both low and high-pass filtering is equivalent to band pass filtering. The direction filtering can be used to remove linear features in the original data (see Appendices G and H). The frequency filtering will be discussed more in the chapter 4.6.

3.3. Process menu

The *Process* menu contains the items for inverse processing tasks. Most of these operations can also be accessed using a quick tool button in the control panel next to the graph area.

Upward continuation	performs upward continuation of potential field data
Downward continuat.	performs downward continuation
Pole reduction	performs reduction to magnetic pole
Equator reduction	performs reduction to magnetic equator
dF/dz gradient	vertical gradient of the data
dF/dx gradient	horizontal x gradient of the data
dF/dy gradient	horizontal y gradient of the data
d^2F/dz^2 *gradient*	second vertical gradient of the data
d^2F/dx^2 *gradient*	second horizontal x gradient of the data
d^2F/dy^2 *gradient*	second horizontal y gradient of the data
$d^2F/dxdy$ *gradient*	xy gradient of the data
$d^2F/dxdz$ *gradient*	xz gradient of the data
$d^2F/dydz$ *gradient*	yz gradient of the data
$(d^2F/dx^2 - d^2F/dx^2)/2$	tensor component measured by Falcon gravity gradient system
Rot. invariant 1	first rotational invariant (gravity tensor)
Rot. invariant 2	second rotational invariant (gravity tensor)
Hori. gradient	horizontal gradient
Total gradient	total gradient (analytic signal)
Tilt gradient	tilt gradient

Max hori. grad	maximum horizontal gradient amplitude
Pseudo gravity	pseudo-gravity field from magnetic data
Pseudo magnetic	pseudo-magnetic field from gravity data
Gravity potential	gravity potential from gravity field (Bouguer data)
Magnetic potential	magnetic potential from total field (TMI) data
Hilbert transform	Hilbert transform (phase shift)
Sun shading	sun shading
General derivative	generalized derivative operator (by G.F.Cooper)
Undo process	reverts back to unprocessed (but frequency filtered) data.

3.3.1. Upward and downward continuation

Upward continuation is an operation that shifts the data by a constant height level above the surface of the earth (or the plane of measurements). It is used to estimate the large scale or regional (low frequency or long wave length) trends of the data. To some degree upward continuation is equivalent to low-pass filtering, since it removes high frequency contents of the data. On the contrary, downward continuation shifts the data below the plane of measurements (e.g. inside the earth). It is used to enhance the high frequency content of the data and to estimate the depth to the top of the targets. The upward and downward continuation can be formulated as (Blakely, 1995):

$$F[U_u] = F[U] \times e^{-\Delta z \cdot k} \qquad \text{and} \qquad F[U_d] = F[U] \times e^{+\Delta z \cdot k}\,, \tag{3}$$

where $F[U]$, $F[U_u]$ and $F[U_d]$ are the Fourier transforms of the potential field U, upward continued field U_u and downward continued field U_d, $\Delta z > 0$ is the elevation difference, and $k = [k_x^2+k_y^2]^{1/2}$ is the radial wave number. The transformed fields are then computed by taking the inverse Fourier transform of $F[U_u]$ and $F[U_d]$. Note that downward continuation is not a stable operation. If the plane of continuation is located below the actual potential field sources the results become erratic (Laplace's equation does not hold anymore). Examples of upward and downward continuation are shown in Appendix I.

Note: The height difference or elevation (Δz) used in upward or downward continuation is read from the *Height* text field in the control panel left to the graph area. The value of height difference is always positive.

3.3.2. Pole and equator reductions

Unlike the gravity field, the static magnetic field of a symmetric body (e.g. vertical prism) exhibits non-symmetric anomaly shape because of the inclined direction of the inducing magnetic field. The behavior of Earth's magnetic field can be estimated using the IGRF model (international geomagnetic reference field). *Pole reduction* transforms magnetic field data as if it were measured on a magnetic pole where inclination is vertical. This helps estimating the true dip and strike directions of the targets. Since pole reduction is an unstable operation at low latitudes *Equator reduction* can be performed instead. These operations involve a phase transformation and they can be formulated as (Blakely, 1995):

$$F[U_p] = F[U] \times \frac{\Theta'_m \Theta'_f}{\Theta_m \Theta_f}, \tag{4}$$

where, Θ_m and Θ_f are functions of the direction vectors of the original magnetization $\mathbf{m}= (m_x,m_y,m_z)$ and external magnetic field $\mathbf{f}= (f_x,f_y,f_z)$, and Θ'_m and Θ'_f are functions of the direction vectors of the transformed magnetization and field ($\mathbf{m'}$ and $\mathbf{f'}$). Unless remanent magnetization is taken into consideration $\mathbf{m} = \mathbf{f}$ and $\mathbf{m}' = \mathbf{f}'$.

Note: The inclination and the declination (angles in degrees) of the original magnetic field (and magnetization) are read from the *Incl.* and *Decl.* text fields in the left control panel. Inclination ($I = \pm 90°$) is taken from horizontal plane and it is positive downwards (northern hemisphere) and negative upwards (southern hemisphere). Declination ($D = \pm 180°$) is taken from the direction of positive y-axis (geographical north) and it is positive in clock-wise orientation. See chapter 4.7 for information about rotating the equator reduced data.

3.3.3. Basic gradients

Vertical (*dU/dz*) and horizontal gradients (*dU/dx* and *dU/dy*) are computed automatically after FFT and kept in compute memory so that other (derived) gradient components can be computed faster. Also second degree gradients are computed automatically and kept in memory so that they can be rotated horizontally faster. Here *x* and *y* directions represents the horizontal axis (East) and vertical axis (North) of the mapped data. The vertical gradients (*dU/dz* for magnetic data and d^2U/dz^2 for gravity data) are particularly useful in sharpening the anomalies below the anomalous sources. The less common gradient components $d^2U/dxdy$, $d^2U/dxdz$, $d^2U/dydz$ correspond to the off-diagonal components of the gravity tensor (g_{xy} , g_{xz} and g_{yz}) provided that they are computed from the gravity potential. Examples of gradient operations are shown in Appendices J and K.

In frequency domain the Fourier transforms of the basic gradients $F[d^nU/dx^n]$ are computed by multiplying the Fourier transform of the field $F[U]$ with wave numbers (k_x, k_y and k). The gradient operations can be formulated as (Blakely, 1995)

$$F\left[\frac{dU}{dx}\right] = ik_xF[U]\ , \qquad F\left[\frac{dU}{dy}\right] = ik_yF[U] \qquad \text{and } F\left[\frac{dU}{dz}\right] = |k|F[U] \qquad (5)$$

$$F\left[\frac{d^2U}{dx^2}\right] = -k_xF[U]\ , \qquad F\left[\frac{d^2U}{dy^2}\right] = -k_yF[U] \qquad \text{and } F\left[\frac{d^2U}{dz^2}\right] = |k|^2F[U] \qquad (6)$$

and

$$F\left[\frac{d^2U}{dxdy}\right] = -k_xk_yF[U]\ , \quad F\left[\frac{d^2U}{dxdz}\right] = -k_x|k|F[U] \quad \text{and } F\left[\frac{d^2U}{dydz}\right] = -k_y|k|F[U] \quad (7)$$

where n= 1 or 2 is the degree of the gradient and i is the imaginary number (i^2= -1). The above mentioned basic gradients are also used to compute additional gravity tensor related components namely the first and second rotational invariants (see Pedersen & Rasmussen, 1990) and the 2.nd Falcon gravity gradient component (g_{xx} - g_{yy})/2.

3.3.4. Derived gradients

The derived gradient operations include horizontal gradient *H*, total gradient *A* (sometimes also called analytical signal), tilt gradient *T*, maximum horizontal gradient amplitude *MH* (also called total horizontal derivative), horizontal gradient of tilt gradient *S*, and theta map *TM* (Blakely, 1995 and Cooper & Cowan, 2006)

$$H = \sqrt{\left(\frac{dU}{dx}\right)^2 + \left(\frac{dU}{dy}\right)^2} \tag{8}$$

$$A = \sqrt{\left(\frac{dU}{dx}\right)^2 + \left(\frac{dU}{dy}\right)^2 + \left(\frac{dU}{dz}\right)^2} \tag{9}$$

$$T = \tan^{-1} \sqrt{\frac{\frac{dU}{dz}}{H(x,y)}} \tag{10}$$

$$MH = Re\left[\tanh^{-1} \sqrt{\frac{\frac{dU}{dz}}{H(x,y)}}\right] \tag{11}$$

$$S = \sqrt{\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2}} \tag{12}$$

$$TM = \tan^{-1} \sqrt{\frac{H(x,y)}{\left|\frac{dU}{dz}\right|}}\,. \tag{13}$$

The derived gradients are used to enhance small amplitude features in the data, to delineate and locate geological targets and contacts, and derive information about structural geology. Examples of derived gradient operations are shown in Appendix L and M.

3.3.5. Potentials and pseudo-fields

Potential fields are defined as the (negative) gradient of corresponding scalar potential (**F**= $-\nabla\phi = -d\phi/dx - d\phi/dy - d\phi/dz$). In Fourier domain the gradients of the field were computed by simple multiplication (Eqs. 5-7). Thus, the potential can be obtained from the vertical component of the field by dividing its Fourier transform by the radial wave number (k). In case of the magnetic potential the direction of the inducing field must be taken into account. Furthermore, the gravity and magnetic potentials (and hence the fields) of an anomalous target with the same dimensions but different petrophysical properties (density and magnetic susceptibility) are interrelated via Poisson's relation. Thus, it is possible to compute so-called

pseudo-gravity field from the measured (anomalous) magnetic data and (anomalous) pseudo-magnetic field from gravity data. The pseudo-gravity and magnetic fields (g_{ps} and ΔT_{ps}) and the gravity and magnetic potentials (ϕ and V) are computed from following equations (Blakely, 1995)

$$F[g_{ps}] = \frac{A}{|k|} \times F[\Delta T_p] \quad \text{and} \quad F[\Delta T_{ps}] = \frac{|k|}{A} F[\Theta_m \Theta_f] \times F[g_z] \tag{14}$$

$$F[\phi] = \frac{1}{|k|} \times F[g_z] \quad \text{and} \quad F[V] = -\frac{1}{\Theta_f |k|} \times F[\Delta T]\,, \tag{15}$$

where ΔT_p is the anomalous magnetic field reduced to the pole, g_z is the vertical component of the gravity field, and A is a function that depends on the density contrast and the intensity of magnetization which in turn is a function of magnetic susceptibility and intensity of the inducing magnetic field. The values of inclination and declination defined in GUI window are used for the direction of magnetization and magnetic field ($\Theta_m = \Theta_f$). The program asks the user for the values of density contrast (g/cm^3), susceptibility (SI) and intensity of the inducing magnetic field (nT). Examples of gravity potential and pseudo-magnetic field are shown in Appendix N.

3.3.6. Hilbert transform

Hilbert transform is a phase-shift operation which is related to the concept of analytic signal. It is usually performed on magnetic data, because the analytic signal does not depend on the direction of magnetization. Hilbert transform is closely related to pole reduction although it is much easier to compute. For a 3-D case analytic signal is defined as a vector field (Blakely, 1995)

$$\mathbf{A}(x, y, z) = \frac{dU}{dx}\mathbf{i} + \frac{dU}{dy}\mathbf{j} + i\frac{dU}{dz}\mathbf{k} \tag{17}$$

where the real and imaginary parts are a Hilbert transform pair. Note that the norm $A = |\mathbf{A}|$ is equal to the total gradient in Eq. (9). According to Blakely (1995) the Fourier transform of (1-D) analytic signal can be computed as

$$F[A] = F[U](1 + \operatorname{sgn}(k)) \tag{18}$$

where sgn(k)=-1 if $k < 0$ and sgn(k)=+1 if $k > 0$. In practice, the 2-D Hilbert transform is accomplished by multiplying the spectrum with two and nulling all items in the lower left quarter of the (centered) amplitude spectrum ($k_x < 0$ and $k_y < 0$). When initiated, the program asks the user if the transform is made for 2-D map data or 1-D trace data. In the latter case the program also asks if the transform is made along x axis (half spectrum $k_x < 0$ is nulled) or y axis (half spectrum $k_y < 0$ is nulled). Note that in FOURPOT the results are returned as the real part of the inverse FFT. In case of Hilbert transform, however, the program will ask the user if the results are given as the absolute of the complex value, which will yield the so-called envelope function of the original function.

3.3.7. Sun shading and generalized derivative

Sun shading and generalized derivative are Fourier operations that can be used to enhance small and linear features in the data. This is accomplished by altering the direction of illumination (or gradient direction) defined by two angles, elevation from the vertical axis (zenith) and azimuth from the positive x axis. They are defined by equations (Cooper & Cowan, 2011)

$$R = \frac{1+p_0 p+q_0 q}{\sqrt{1+p^2+q^2}+\sqrt{1+p_0^2+q_0^2}} \quad \text{and} \quad GD = \frac{(p\cdot\sin\theta+q\cdot\cos\theta)\cos\phi+p\cdot\sin\phi}{A(x,y)}, \qquad (19)$$

where p_0=-cosθtanϕ, q_0=-sinθtanϕ , $p=dU/dx$ and $q=dU/dy$ and θ and ϕ are angles that define the azimuth and elevation of the illumination (or gradient direction), and $A(x,y)$ is the total gradient defined in Eq (9). Examples of gravity sun shading and generalized derivative are shown in Appendix O.

<u>Note</u>: The elevation and the azimuth (angles in degrees) of the sun shading and generalized derivative are read from the same two text field as the inclination and declination. Elevation (0-180°) is taken from the zenith (out of the plane of the mapped data) and azimuth(±180°) is taken from positive x-axis (east) and it is positive in clock-wise orientation. See chapter 4.7 for more information about rotating the azimuth direction.

Undo processing menu item can be used to revert back to the unprocessed status. Possible low-pass, high-pass or directional filtering operations, however, will not be unmade.

3.4. Tools menu

The *Tools* menu contains following items:

Median filter 3x3	smooth the data using a median of a moving 3x3 window
Ridge tools	pick continuous ridges, fingerprint directions and remove them
Profile/Model tools	pick a profile and create initial 2.5D prism model automatically
Radial spectrum	show the radial spectrum used for depth analysis

Median filter 3x3 can be used to smooth gridded data and to remove outliers (spikes) from it. Median filter can often greatly improve the results of many Fourier operations which assume that the data and its derivatives are continuous. The *Pick ridges 3x3* and *Pick ridges 5x5* menu items utilize an experimental algorithm that investigates if the grid point at the center of a moving 3×3 or 5×5 window represents a continuous ridge-like anomaly peak. Ridge is defined by three coordinate pairs; the first indicates the center of the ridge, the second and the third indicate where the ridge comes from and where it goes to. The ridges can be straight (e.g. W-E or SE-NW) or bent up to 90 degrees (eq. W-SE or SE-NE). Ridge points can be saved into a column formatted text file using *File/Save ridge* data menu item. The ridge data can also be saved in Atlas BNA format.

<u>Hint:</u> Ridge-picking is usually applied on horizontal gradient data to locate geological contacts. When the operation is re-applied on upward continued data one can estimate the dip directions of the contacts. To do this one should save the results of the ridge picking made on horizontal gradient of original data in BNA format, perform upward continuation, apply Inverse→data menu item, perform the ridge picking on the upward continued data and read in the previous BNA file for comparison.

Pick directions is somewhat similar operation that computes the orientation or principal axis directions of the data based on the x and y gradients of the data. Directions are saved as angles in degrees taken from the horizontal x axis (east) towards positive y axis (north). Both

ridge-picking and computation of principal directions ask the user for a threshold value (in percent) that will ignore smaller features when increased.

The items in the *Profile/Model tools* and *Radial spectrum* sub-menus are discussed in chapters 4.9 and 4.10.

3.5. Exit menu

Exit menu has two items. The first one restarts the whole program GUI window so that it suits better either normal 4:3 or widescreen displays. When changing to widescreen mode the program asks the user for a scaling value for the aspect ratio. Value 1.0 is used for 4:3 displays and values 0.6-0.8 should be used for widescreen displays.

OK to Exit item will close the program without any additional warnings. Therefore, the user should take care to save the results before exiting. Errors that are encountered before the GUI starts up can be found from the FOURPOT.ERR file. When operating in GUI mode, run-time errors arising from improper parameter values, for example, are shown on the screen.

4. Using the program

When the program is started it reads graph parameters and some additional settings from the FOURPOT.DIS file (see chapter 5.4 for more information). If this file does not exist, default parameters will be assigned and the file is created automatically. The program then builds up the GUI shown in Appendix A. None of the program controls (widgets), however, are active before data has been read in.

The data processing and analysis is performed in few successive steps that are discussed next. The preliminary step is always to check that the input data file is in correct format so that it can be read in (see chapter 5 on file formats). After reading in the data one should apply a) *Make grid*, b) *Padding* and c) *Plot FFT* buttons to perform interpolation and padding and to compute the 2-D FFT and its inverse. Only after the FFT has been computed one can perform frequency filtering (*Edit* menu) and other frequency domain data processing (*Process* menu) and take a look at the inverse (*Plot inverse*) and difference (*Plot difference*).

4.1. Reading in the data

The first thing to do after starting up the program is to read in the data using the *Read data* (*.DAT), *Read grid* (*.ASC) or *Read matrix* (*.MAT) menu items. Note that Geosoft XYZ formatted files can be read as normal DAT files provided that the file header is suitable. Also note any missing data values inside ASC files are replaced for data processing by the median of the special points and blanked out by white color in the map graphs. See chapter 5 for more information about file formats.

After successful data input the program activates *Plot data* and *Reset* buttons and plots a contour (or image map) as seen in Appendix A. If the data were read from a DAT file it is assumed to be irregularly sampled and the *Make grid* button is active. If the data were read from an ASC file the data are already regularly sampled and *Make grid* is inactive (until Reset button is pressed). The initial grid sampling is based on the number of data values and the spacing between the first two data points in the file. Depending of the current status of *Step number/distance* option in *Edit/Miscellaneous* submenu, the number of grid nodes (*N* and *M*) or the distance between nodes (*dx* and *dy*) will appear in *Step x* and *Step y* text fields in the left control panel.

4.2. Interpolation

All input data read from DAT files are assumed to be irregularly sampled. Even if the original data were already evenly discretized it will be re-interpolated before it's passed to FFT. Therefore, after reading in the data the user must first provide suitable values for the number of grid points (*N* and *M*) or grid stepping (*dx* and *dy*) in the *Step x* and *Step y* text fields and then press the *Make grid* button to perform the required interpolation. Note that the grid spacing (*dx* and *dy*) does not need to be equal in x and y directions.

After interpolation, *Make grid* button becomes inactive and *Plot data* button is used to visualize the original, automatically interpolated data. To define new grid sampling one must first revert back to original (automatically computed) sampling using the *Reset* button, edit new values for the *Step x* and *Step y* text fields and press the (newly active) *Make grid* button. Note that normally *Step x* and *Step y* text fields show the number of nodes (*N* and *M*). Only when looking at the original data (*Plot data* button) and when *Step distance* option (*Edit/Miscellaneous*) is active they can show the grid node step distances (*dx* and *dy*).

<u>Note:</u> The interpolation discussed above is performed using the DISLIN subroutine GETMAT (see DISLIN user manual). This subroutine works best if the data are already regularly sampled and the new grid coincides with the original grid. If the grid sampling is too dense compared to data sampling the interpolation does not give good results and a blank (white) map may be shown. In this case the user should provide larger step values. Alternatively, highly irregular data should be interpolated on a regular grid using more advanced interpolation algorithm (e.g, minimum curvature) of third party software (e.g., Golden Software Surfer).

4.3. Padding and tapering

Padding and tapering are essential parts of successful Fourier transform processing. The *Padding* button will automatically add extra columns and rows around the interpolated data matrix so that the grid dimensions (*N* and *M*) will become even powers of two (e.g., 64, 128, 256, 512, etc.) that is required by the FFT algorithm. The padding can be performed with or without tapering. Tapering means that artificial values are added to the padded area so that the

continuity of the data is preserved at the edges. See Fig. 1 and Appendix B for examples of padding.

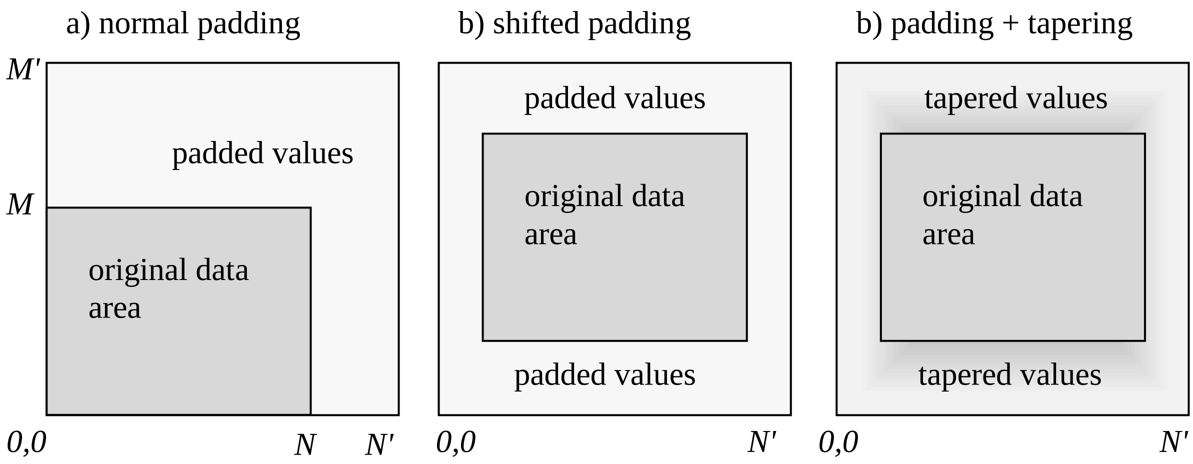

Figure 1. Schematic view of a) padding without shift, b) padding with shift and c) shifted padding with tapering. The new dimensions *N'* and *M'* are powers of two.

Sometimes the original grid dimension may already be so close to some power of two that the tapering will not ensure enough data continuity. To further increase the padded dimension from 64 to 128, for example, one needs to change the current values of *Step X* or *Step Y* to some number between 65 and 128 and then press the *Padding* button again. Note that the dimensions of the grid can be decreased only using re-interpolation (*Make grid* button).

Padding and tapering depend on the selection made with the *Padding mode* item in *Edit* menu. The padding modes are: a) *Zeros,* which actually uses for padding the median of few selected data points instead of zeros, b) *Linear extrapolation*, which will extend with the outmost data point, c) *Mean value based, which* will use the nearest points inside some increasing search radius and d) *Gradient based*, which uses the mean value and the derivative of the field to extend the data to the padding zone. The *Shift yes/no* option is used to define if padding is added only to the top and to the right of original data area or if padding is made all around the data area (which looks like the original data area were shifted).

<u>Note:</u> The best results are usually obtained using gradient based padding with shift. The other padding options are preserved primarily for testing and teaching purposes so that the users can see the effect and importance of padding and tapering.

4.4. Fast Fourier transform

After sampling, interpolation and padding the user must press the *Plot FFT* button to compute the FFT and to see the 2-D frequency spectrum (see Appendix C). When *Plot FFT* button is pressed (if valid inverse results do not exist), the automatic low-pass filtering (if activated) is also made, and the inverse FFT is computed, and inverse results and the difference between original (interpolated) data and inverse transformed data are computed.

Note that the true frequency spectrum F is complex having both real and imaginary parts, $F = \mathrm{Re}(F) + i\cdot\mathrm{Im}(F)$. The FFT graph, however, shows the 10-base logarithm ($\log_{10}$) of the *amplitude spectrum* $A = |F| = [\mathrm{Re}(F)^2 + \mathrm{Im}(F)^2]^{1/2}$. The graph is centered so that the origin ($k_x=k_y=0$) is located at the center and the horizontal and vertical axes range between the wave numbers (k_{Nx}, k_{Ny}) related to the negative and positive Nyquist frequencies (f_{Nx}, f_{Ny}), the values of which are shown in the information text next to the graph. The lowest frequencies (largest wave lengths = widest data variations) are located in the middle of the graph and the highest frequencies (shortest wave lengths) are located far from the origin. Continuous linear features appear as linear features in the FFT spectrum as well (the angle of slope is inversed, though).

4.5. Inverse FFT and difference

Once the FFT has been computed and the 2-D frequency spectrum is available one can perform (non-automatic) frequency filtering and the various frequency domain processing tasks. The *Plot inverse* button will display the (current) inverse results. The *Plot diff.* button will display the difference between the original (gridded) data and the inverse data. If automatic low-pass filtering is not active, the *Plot inverse* will show the inverse of the original FFT and the difference will be very close to zero. If automatic low-pass filtering is active, the inverse results show a smoothed version of the original map and the difference will show the high-pass filtered parts of the data. When processing tasks are made (*Process* menu or quick task buttons) the inverse results will be shown automatically, i.e., without pressing *Plot inverse* button.

Once the gridding has been made the *Plot data* button will show the original (interpolated). Successive utilization of *Plot inverse* and *Plot data* buttons can be used to visualize the changes that are made to the data due to the frequency domain operations. As an alternative,

the *Plot diff* button can be used to visualize the difference between the original (interpolated) data and the inverse transformed data. Note that pressing the *Reset* or *Padding* buttons will invalidate the FFT and current inverse results and the user needs to perform the frequency filtering operations again. Note also that after interpolation the *Make grid* button becomes inactive, because the *N* and *M* are affected by the padding to the next power of two. To be able to re-grid the original data one needs to apply the *Reset* button.

Note: The Fourier operations are made on the interpolated and padded data matrix. Sequential Fourier operations (e.g. first reduction to pole and then vertical gradient) can be performed using the *Edit*/*Inverse→Data* menu item. In this case the data are already defined on a regular grid so the process can be continued from padding step and the *Plot data* button reverts back to the original data. Alternatively, one can save the current inverse results and read the file as new input data.

4.6. Frequency filtering

Frequency filtering affects the current FFT spectrum on which gradients components and all other processing functions are computed. This means that frequency filtered data are always passed to further processing operations without the use of *Inverse→data* menu item. As a matter of fact, low-pass filtering is often an essential preliminary step for successful operation of the gradient filters.

Frequency filtering means that some parts of the spectrum are nulled. Rapid changes in the spectrum, however, can cause oscillations in the inverse transformed data. This is known as the Gibbs phenomenon. Therefore, instead of using rectangular or box-car shaped filter functions, bell shaped smoothing is applied over the cut-off range of the filter. In low-pass and high-pass filtering this means in practice that instead of a single filter ring the user must provide the radii of the inner and the outer ring of the cut-off range. Between the two rings half-sine or half-cosine function is used to define the smoothing. In directional filtering the user needs to provide the direction (angles) of the upper and lower limit of the (notch) filters and the width (angle) of the cut-off range. The filter rings and sectors and the cut-off ranges are illustrated in Figure 2.

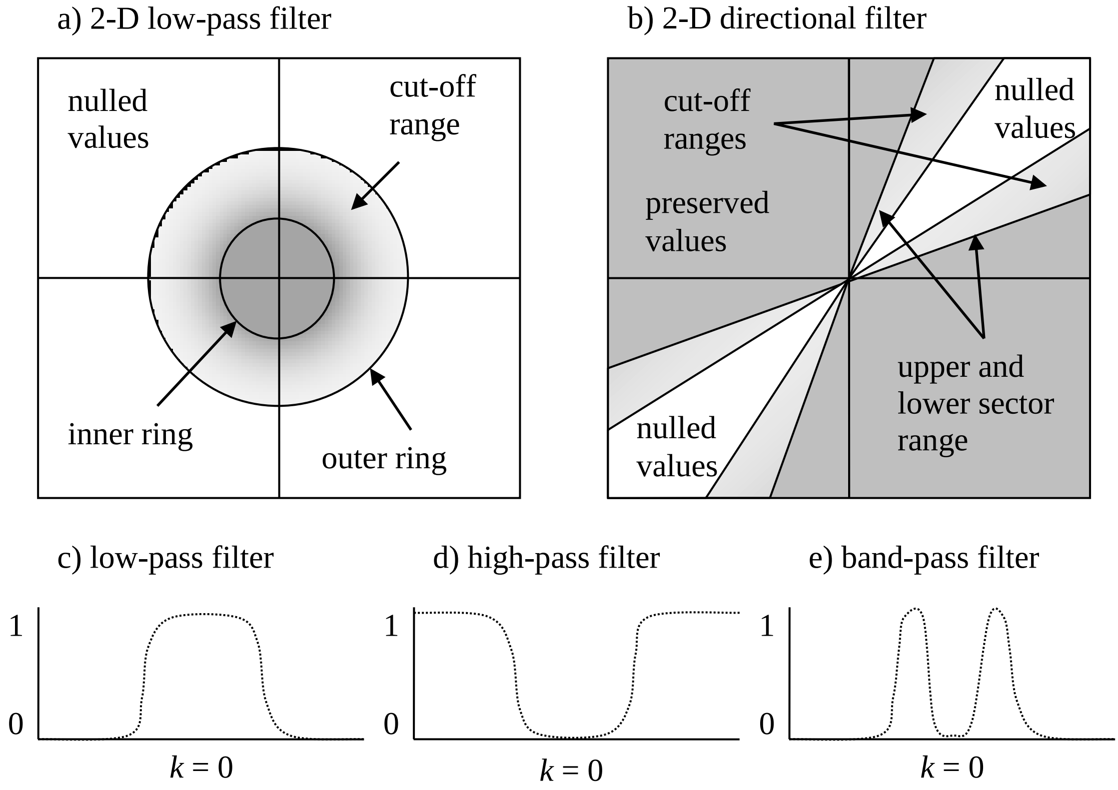

Figure 2. Schematic view of a) cut-off rings and b) sectors of 2-D filters and the smooth (bell shape) filter function of c) low-pass, d) high-pass and e) band-pass filters.

4.6.1. Low-pass and high-pass filtering

In low-pass filtering all data outside the outer ring (high frequencies) are nulled and all data inside the inner ring are preserved (see Fig. 2a and 2c). On the contrary, in high-pass filtering all data inside the inner ring (low frequencies) are nulled and all data outside the outer ring are preserved (see Fig. 2d). The radii of the rings are defined as wave length (λ), which is the inverse of radial frequency $k= [k_x^2+k_y^2]^{1/2}$, where k_x and k_y are the axes of the 2-D frequency graph.

Note: The x and y axes of the 2-D FFT plot are not scaled. If sampling (*dx* and *dy*) is different in the x and y directions the Nyquist frequencies, i.e., the minimum and maximum values of the axes are different. In the present version of FOURPOT, the frequency filtering uses the same wave length both in x and y directions. The lengths of the k_x and k_y axes, however, are the same. As a consequence, circular filter rings will appear as ellipses in the 2-D FFT plot.

4.6.2. Directional filtering

Directional filtering is equal to the *fk*-filtering commonly used in seismic and GPR data processing. Elongated, linear features in the data appear as linear features also in the FFT spectrum. The slopes are reciprocal, so that horizontal features (parallel to x axis) in the data are shown close to the vertical k_y axis in FFT spectrum. Directional filter removes linear features from the data by nulling frequency content inside given sector (see Fig. 2b). The directional filter is defined by two angles that correspond either a) to the upper and lower limit of the sector (interactive mode) or b) to the center and the width of the sector (manual mode). The angles are defined in degrees in counter-clockwise direction taken from the positive k_x axis. Because of the symmetry of the frequency spectrum the filter will be automatically mirrored around the origin. Thus, the user needs to define a single sector either above the k_x axis or left to the k_y axis. In addition, the width of the cut-off range must be defined by providing a third angle.

4.6.3. Interactive and manual filtering

Frequency filtering can be done either a) interactively using the mouse or b) manually by providing the numerical values of the filter cut-off ranges. The selection is made when the corresponding menu item is chosen in the *Edit* menu.

When interactive mode is chosen most of the other program widgets become inactive and the mouse cursor changes from normal arrow into a cross-hair above the graph area (note: only above the graph area). The 2-D frequency spectrum is shown together with auxiliary polar coordinate grid. The user is given instructions to provide first the inner and then the outer ring by pressing the left mouse button above the graph. To validate the last value and to end the editing mode one must press the right mouse button. The distance from the origin determines the radius of the filter rings. Likewise in directional filtering the user is instructed to provide the upper and lower angle of the filter sector and the width of the cut-off range. The width can be defined as an angular difference from the lower or upper sector and the filter cut-off range will be adjusted symmetrically at both sides automatically.

Note that the ring and sector values can be given in reverse order in which case they will be automatically arranged properly. Values can be omitted by pressing the right mouse button

instead of the left one. In low-pass and high-pass filtering omitting the first value cancels the whole interactive editing mode. Omitting the second value will make the two rings the same which means that the sine/cosine bell function is replaced by a box-car filter. In directional filtering the upper and lower angle must always be provided but the cut-off range can be omitted if the right mouse button is pressed when its value is asked for.

Note: The filter rings and sectors will be shown by dotted lines on the frequency spectrum. In addition, the wave lengths corresponding to inner and outer ring and the limit angles of the sector are shown on the information text of the graph.

After the user gets accustomed with interactive filtering one should learn manual number-based filtering which is more accurate than interactive filtering. Manual filtering is made by providing the wave lengths (in units of dimension) of the filter rings (low-pass and high-pass filtering) in the input dialog that appears after the filtering is initiated from the *Edit* menu.

4.6.4. Automatic low-pass filtering

The results of most frequency domain filtering operations (especially the second degree gradients) are strongly affected by the "smoothness" and continuity of the data. To circumvent computational artifacts in the inverse transformed data it is often necessary to perform low-pass filtering before any other processing operations. *Automatic LP filter* item enables/disables automatic low-pass filtering. To activate automatic low-pass filtering the user needs to provide the radii of the filter rings manually. Note that in this case the inner and outer radii are defined as a frequencies normalized by the maximum Nyquist frequency (i.e., a value between 0 and 1) in both x and y directions (not as wave lengths). Thus, low-pass filter with default values 0.5 and 0.67 removes frequencies greater than two thirds of Nyquist frequency (=0,67*0,5/dx) or wave lengths less than 3*dx also in y direction if $dx < dy$. Automatic values are used if both values are zero. The automatic low-pass filtering is disabled if the input values are omitted totally, i.e., a blank line is given as input.

4.7. Rotation and transparent shading

Horizontal rotation around z axis can be applied to basic gradient components as well as reduction to equator, sun shading and generalized derivative operations to enhance linear features of varying direction in the potential field data. The *Activate rotation* check box must first be activated and a non-zero value provided for the *Rotation angle* text field. The *Rotate* button is then used to add one rotation step to the current value of total rotation. Rotation is not made if the current view is not one of the abovementioned inverse results. Depending on the current view the information text on the graph shows either the total rotation of 1.st degree gradient or 2.nd degree tensor component, rotated field direction of equator reduction or the azimuthal direction of sun-shading or generalized derivative. The *Reset rotation* button resets the total rotation to zero.

When basic gradients (1.st and 2.nd degree gradients) are rotated their values are updated in computer memory so that all derived components can be computed faster. However, except for the (Gxx-Gyy)/2 (Falcon response) none of the derived gradient components depend on horizontal rotation. After resetting the x and y gradients are again directed along x and y axes of the data.

In pole reduction the transformed field becomes vertical **m'**=**f′**= (0,0,1) and declination does not play any role. Normally in equator reduction the transformed magnetization and field will be rotated towards north (**m'**=**f′**= (0,1,0)) based to the given value of declination. However, the direction of the transformed field reduced to equator can be rotated. Thus, for example, the reduced field can be computed in EW-direction using an angle of ±90° (**m'**=**f′**= (±1,0,0)).

When performing sun shading and generalized derivative, the elevation and azimuth angles are read from the same two text fields as the inclination and declination used in pole and equator reduction and pseudo-field computations. Instead of editing the azimuth angle manually, the *Rotate* button allows fast rotation of the direction of sun shading and generalized derivative. Resetting makes the azimuth zero (along x axis).

Most of the other gradient operators as well as sun shading and generalized derivative are more informative when displayed using greyscale colors. Activating the *Transparent shading* check box allows displaying the inverse results as greyscale colors transparently below the

original (unprocessed but gridded) data that uses the current (user selected) color scale. When activated the program asks the user for the strength of transparency. Transparency ranges between 0 (fully transparent) and ±1 (fully opaque). Negative value of transparency will apply inverse greyscale. Examples of transparent color shading are shown in Appendix P for sun shading and generalized derivative.

Together with transparent shading the rotation can be used to find the direction or directions that enhance wanted features in the data. Remember, however, that some gradient components and derived gradients are invariant to horizontal rotation. Also note that some inverse results (pole and equator reduction, potentials and pseudo-fields and Hilbert transform) are not shown above the original data transparently. Instead the inverse results are shown transparently on top of themselves. Although this may sound unnecessary, it often improves the visibility of data maxima or minima depending on the current color scale.

4.8. Color mapping (range, center and levels)

Range and *Center* scale widgets can be used to change the color mapping of the data maps and 2-D frequency spectrum. *Levels* widget is used to increase or decrease the number of contour levels (default is 21).

By default the color scale (e.g. rainbow or reverse rainbow) is evenly distributed between the minimum and the maximum data values of the current map. *Range* scale widget changes the width of the color scale between 5 % and 125 % of the original minimum and maximum values. Decreasing the value is useful if the maximum data value of the map is larger than the mean data level due to outlier, for example. Increasing its value is useful if the maps get saturated at their minimum or maximum limit values.

Center scale widget changes the middle point of the color scale between -85 % and +85 % of the current data range. Increasing the center value will emphasize small data values and increases the amount of colors at the beginning part of the color scale. Decreasing the center value will emphasize large data values and increases the amount of colors at the end of the color scale.

Note: After changing any of the abovementioned scales the user must press the *Update* quick button to make the changes effective. Because the scale widgets give slightly different results for different graphs one can reset the color mapping parameters to their defaults using the *Reset* quick button.

Note that FOURPOT was not designed to be used as a plotting program. The user is advised to save the mapped results into text files and use more advanced programs like Golden Software's Surfer for publication quality plotting.

4.9. Profile/Model tools

The six push buttons below the *Levels* scale widget at the bottom of the left control panel are used a) to see the results of various filtering operations as a line graph along a straight user defined profile and b) to create an initial model based on 3-D dipping prisms for the interpretation of the potential field data along the picked profile. These tasks are also available through the *Tools/Profile/Model tools* sub-menu.

4.9.1 Profile plotting

Pick prof button is used to define the x and y coordinates of the profile start and end. This can be done either interactively with the mouse or more precisely giving the coordinates in an input dialog. After applying the *Pick prof* button the gridded (and frequency filtered) data is interpolated (bilinear method) along the profile and shown in the graph area. An example of the profile response is shown in Figure 3.

Unless some processing operation has already been made the profile graph shows only the unprocessed (frequency filtered) inverse data against the y axis on the left. Processed data (e.g. vertical gradient in Fig. 3) are shown against the auxiliary y axis on the right. The profile output thus allows detailed comparison of the effects of various processing operations. If the user changes the view form profile graph to 2D map (i.e., using Plot data, Plot inverse or Plot FFT buttons) one must use the *Show prof* button to see the profile response again.

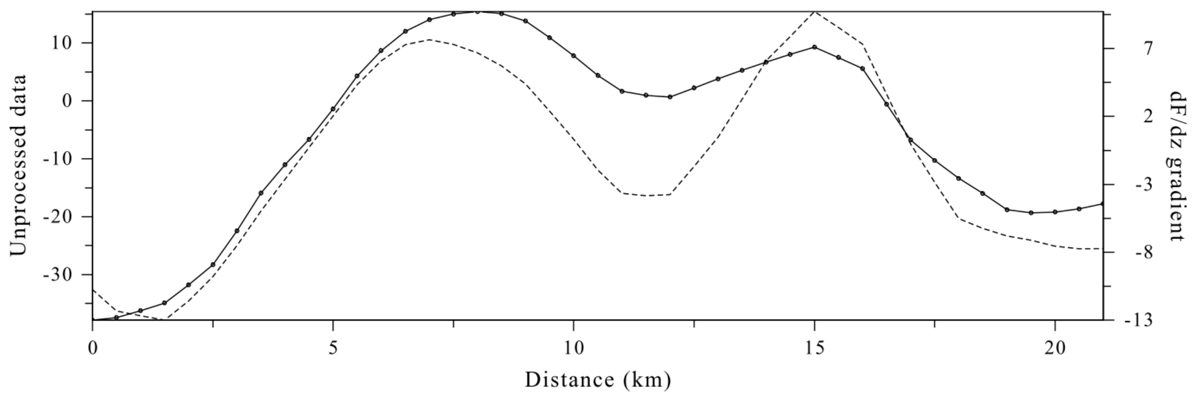

Figure 3. Example of the profile response. The solid and dashed curve correspond to the unprocessed (frequency filtered) data (left y axis) and the first vertical derivative (right y axis) interpolated along the profile.

Once the profile has been generated, the data along it can be saved into column formatted text file using *File/Save profile data* menu item. The file will contain lots of information: columns 1-4 contain xyz coordinates and running profile coordinate (distance); columns 5-12 contain the base level (discussed later), unprocessed data, the first xyz gradients, horizontal gradient, total gradient, tilt gradient and the last column contains whatever processed data is currently active. When profile data is saved the program asks if the data are saved with or without the GRABODS/MAGBODS header information (discussed later).

4.9.2 Prism model generation

Once profile data has been saved to disk one could start model based inversion to estimate, for example, the density contrast or magnetic susceptibility, depth extent and dip angle of the geological targets giving rise to potential field anomalies. The tilt gradient, however, contains useful information that can be used a) to locate the number and the center position of the anomalous targets and b) to divide the data into sub-anomalies, i.e., sections of data the inversion should use per each model prism . FOURPOT uses the tilt gradient data to generate a semi-automatic model for GRABODS and MAGBODS modeling and inversion software by M. Pirttijärvi. These programs, which are still under development and have not been put to public distribution, can be used to interpret single profile data using multiple dipping prism models.

To create the initial model the user must first define the regional field i.e. the base anomaly for the profile. This is made pressing *Edit base* button. Once the interactive mode is entered the user needs to press the left mouse button over the response graph to define the knot points through which the base anomaly should go. The editing mode is ended with a single click of right mouse button. If more than two base knot points are defined cubic splines are used to compute the regional trend (see Figure 4). If only one or two knot points are defined the base anomaly will be a (horizontal or dipping) line. Existing base anomaly can be removed pressing the left mouse button once outside (to the left or to the right) of the graph area. New knots can be added to existing base anomaly pressing the left mouse button once either above or below the profile graph and then once at the new knot location over the graph. Likewise, knots can be removed from the base anomaly pressing the left mouse button once over the graphs close to the selected point and then once above or below the response graph.

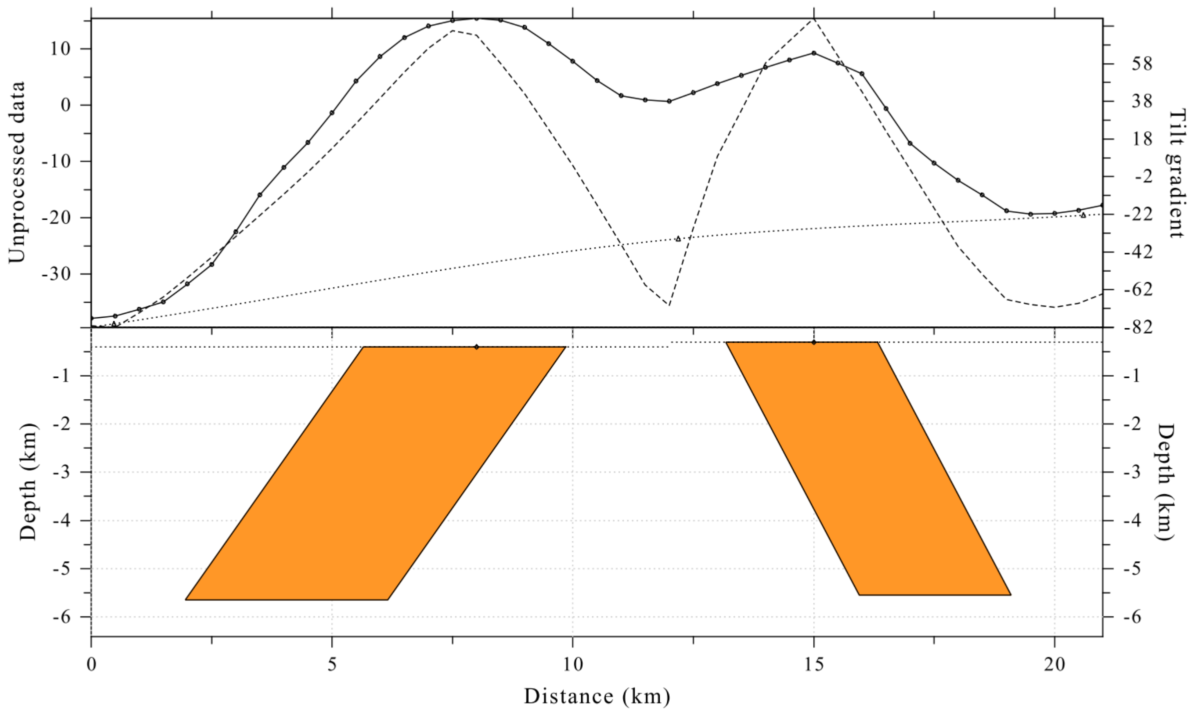

Figure 4. Example of the prism model generated using tilt gradient data. The solid and dashed curves are the unprocessed (frequency filtered) and the tilt gradient interpolated along the profile. The dotted line is the base level fitted using splines through the three base level knots. The dotted horizontal line at the top of prisms is the width of the sub-areas to be used in the (GRABODS) inversion algorithm.

Once the base anomaly has been defined the *Make mod* button for the automatic prism model becomes active. The number and location of the prism models is based on the positive peaks of the tilt gradient data. The tilt gradient varies between -90 and +90 degrees. Usually positive potential field anomalies are reflected as positive values of tilt gradient and its sign becomes negative between two anomalies. The only input parameter needed for model generation is a threshold value which can be used to ignore small anomalies. The threshold value depends on the tilt gradient data but usually the default value of 0.0 is a safe choice. Reducing the threshold below zero allows smaller anomalies to be added to model generation. Vice versa, increasing the value rejects small anomalies and builds model only below the strongest anomalies. When the model has been generated a vertical cross-section of the model will appear below the response graph (see Figure 4).

The model generation uses the minima around the positive peaks of the tilt gradient data to define the sub-areas for the inversion (dotted lines through the top of the prisms in Figure 4). The width of the sub-areas is then used to define the initial value for the thickness of the prism models and the anti-symmetry of the sub-areas (with respect to the center of the bodies) is used to define the initial dip angle. The strike extent (width of the prism models in a direction perpendicular to the profile) and depth extent are based on length of the profile. The density or magnetic susceptibility is based on the amplitude of the potential field data at the center location.

The model can be saved into MAGBODS or GRABODS input file format using *File/Save model file* menu item. For this purpose the data type needs to be set accordingly using *Edit/Data type* sub-menu. The user should also take care to define the spatial dimension correctly using *Miscellaneous/Meters/Kilometers* menu item. When the model is saved the user is given an option to save the profile data as discussed in chapter 4.9.1.

4.10. Radial amplitude spectrum

The items in the *Tools/Radial spectrum* submenu become operational when the 2D Fourier spectrum is the active graph. *Show spectrum* item shows the amplitude spectrum as a function of radial wave-number k_r. The radial ampl7itude is computed as the mean of the 2-D Fourier amplitude spectrum $A= |F|= [\mathrm{Re}(F)^2+\mathrm{Im}(F)^2]^{1/2}$ along rings with radius $k_r= [k_x^2+k_y^2]^{1/2}$ centered on the origin (k_x=k_y= 0). The amplitude spectrum has traditionally been used to

estimate the depth to the top of the potential field sources. This is accomplished by fitting linear lines to the decaying amplitude curve on a semi-logarithmic scale. An example is shown in Figure 5.

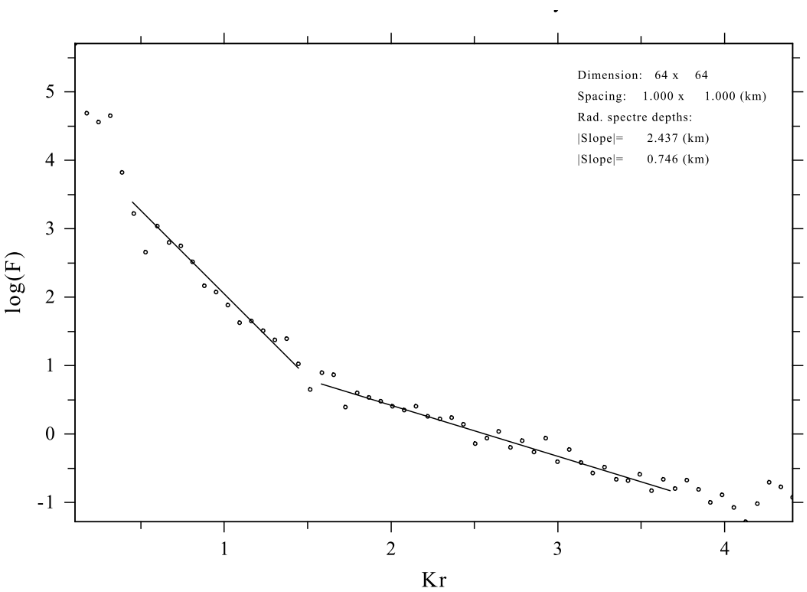

Figure 5. Radial amplitude spectrum with two manually fitted lines. The slopes of the lines give estimates for the depth of the potential field sources. The spectrum is related to the gravity data shown in the Appendices.

In frequency domain the Fourier transform of a potential field can be formulated as $F \approx Ce^{-hk}$ giving $\log(F/C) = -h \cdot k$. Thus, the depth (h) to the top of the source is related to the tangent of the line that is fitted to the linear parts of the amplitude spectrum (on semi-logarithmic scale). The coefficient C depends on the selected data type (*Edit/Data type*): for generic (undefined) data $C = 1$, for gravity data $C = -1/(k_x k_y k)$, and for magnetic data $C = -1/(k_x k_y)$. Depending on the selected data type (and the data) the tangents give different depth estimates that may or may not be realistic. In such cases one should prefer to the un-normalized spectrum ($C = 1$). Please, be noted that the method used for depth determination has not been verified. Refer to Bhattacharyya (1967) and Ruotoistenmäki (1987) for more information about the use of radial amplitude spectrum in depth determination.

Interactive line fitting is initiated using the *Add new line* item in the *Tools/Radial spectrum* submenu. The normal mouse pointer (arrow) will change into a cross-hair cursor over the graph area and most of the GUI widgets become inactive. The user should then select the start and end points of the line over the radial amplitude spectrum by pressing the left mouse

button twice. In case of multiple mouse clicks the last two points are used. The editing is ended by pressing the right mouse button (once). The graph will be redrawn with the new line on it and the slope of the line will be displayed on the information text on the right side of the graph. Multiple lines can be added to the same graph. The *Delete last line* item removes the most recent line.

Note: Because of the cut-off range, the radial spectrum of low-pass and high-pass filtered data may look quite different from the spectrum of unfiltered data. Also note that the horizontal axis starts from zero and ends either at the (minimum) Nyquist frequency or the outer ring of low-pass filtering. In other words, low-pass filtered part of the spectrum is not shown at all. In this case a vertical dotted line indicates the position of inner cut-off rings of low-pass filter (or high-pass filter if applied).

5. File formats

Data can be read and saved in three file formats: 1) column formatted data file (*.DAT) and 2) ArcInfo ASCII grid file format (*.ASC) and 3) gridded data matrix (*.MAT).

When saving data into file the contents of the current graph will be saved (original data cannot be saved again). Thus, either a) the interpolated data together with the inverse transformed (and processed) data and their difference, or b) the padded (and interpolated) data, or c) the (centered) Fourier transform the will be saved. In the first case the program asks the user if the data will be interpolated (bilinear interpolation) at original data locations or not.

5.1. Column formatted data files

The format of a *.DAT file is illustrated below:

```
4096  1  2  4
   0.00  0.00  -0.8665106E+01  0.33489
  10.00  0.00  -0.8558651E+01  0.44134
  20.00  0.00  -0.8365253E+01  0.63474
...
```

The header line at the first line defines the number of lines (NOP) and the index number of the columns that refer to the x and y coordinates (ICO1, ICO2) and the data (ICO3). The file can contain multiple columns of which only one will be read. FOURPOT ignores empty lines and lines starting with ", /, l, L or # characters. Thus, provided that NOP is large enough and ICO1, ICO2 and ICO3 refer to correct column indices, FOURPOT can also read Geosoft XYZ-files (*.XYZ) illustrated below:

```
5000  1   2   4
/ Test data
LINE 1
    0.00     0.00 -0.8665106E+01  0.33489
   10.00     0.00 -0.8558651E+01  0.44134
   20.00     0.00 -0.8365253E+01  0.63474
...
LINE 2
    0.00   100.00 -0.8165106E+01  0.23489
   10.00   100.00 -0.8258651E+01  0.34134
   20.00   100.00 -0.8465253E+01  0.53474
...
```

If the first line starts with a comment character (either / or #) the user must provide the relevant parameters (NOP, ICO1, ICO2, ICO3) interactively using an input dialog that appears. The example below illustrates the generic column file format:

```
# x y z1 z2
   0.00     0.00 -0.8665106E+01  0.33489
  10.00     0.00 -0.8558651E+01  0.44134
  20.00     0.00 -0.8365253E+01  0.63474
...
```

5.2. ASC grid files

The ArcInfo ASCII grid file (*.ASC) format (a.k.a. ESRI grid) is supported because it is quite common and Golden Software Surfer supports it directly. The example below illustrates the ASC format. Please, see the Wikipedia article (http://en.wikipedia.org/wiki/Esri_grid) for the details of the file format.

```
ncols 141
nrows 241
xllcorner 3097500
yllcorner 6597500
cellsize 5000
nodata_value 1.70141000000000001E+038
54.354520061852583 51.976414969718171 49.685123418514031
48.359720260434152...
...
```

Note: The ASC format requires square grid cells ($dx = dy$). However, FOURPOT saves the data in ASC file even if the x and y steps are not equal. In this case `cellsize` in the header corresponds to grid step in x direction and the user needs to manipulate the y coordinates to get them right. The correct y coordinates are computed as $y' = (y-y_0)*dy'+y_0$, where y_0 is the y coordinate of the origin (`yllcorner`) and $dy'= dy/dx$ is the ratio of the true y step with respect to the x step. Note also that the data are stored as rows from top to bottom but the header defines the x and y coordinates of the lower left corner.

5.3. Regularly gridded matrix files

The matrix format (*.MAT) assumes that the data that are already regularly sampled and stored *without* x and y coordinates. The matrix format is provided for better functionality with

programs such as Matlab and Maple. The format of a matrix file (*.DAT) is illustrated in the example below:

```
   64    64
-0.43160E+09
-0.43320E+09
-0.43270E+09
...
```

The header line defines the number of data points in x and y directions (NP= *N* and MP= *M*). The following (NOP = NP × MP) lines contain the actual data values is a single column.

Since the matrix file does not contain coordinates it is important to know the order in which the data are stored. By default the data is stored in column-wise fashion from bottom to top and from left to right. The origin is always located in the bottom left corner. Using generic programming notation we would have:

```
do i= 1,np
   do j= 1,mp
        read or write f(i,j)
   end do
end do
```

The default order can be changed so that for each column the rows will be read by giving a negative value for the number of points in x direction (NP= -NP). This works with data stored using following do-loop:

```
do j= 1,mp
   do i= 1,np
        read or write f(i,j)
   end do
end do
```

Furthermore, if MP < 0 the data are stored in multiple columns. If NP > 0 and MP < 0 each line is considered to be one y columns of the mapped data (as in the default case). If NP < 0 and MP < 0 the lines are considered to be data rows (as in the ASC-file). Using generic programming notation we would have in this case:

```
do j= 1,mp
    read (f(i,j), i=1,np)
end do
```

Note: Because the matrix format does not contain coordinates the sampling frequencies and corresponding wave lengths shown by FOURPOT are computed after the user provides the necessary values in the dialog window that appears on the screen. The input dialog is used to define the x and y coordinates of the origin and the x and y step between the matrix elements. Note that if the x and/or y steps are negative, the axis gets reversed. Moreover, the data are mirrored in x or y direction if the last two parameters in the dialog are set to one instead of zero. Because normally the origin is considered to be in the lower left corner the negative steps and mirroring allow redefining the order in which the data are stored.

5.4. Graph parameters

Several graph parameters and text strings can be changed by manually editing the FOURPOT.DIS file with a normal text editor. This allows the users to translate (localize) the graphs to suit one's needs better.

Note that the format of the FOURPOT.DIS file is important. If it becomes corrupted FOURPOT is likely to crash when started. In this case one should delete the file and a new one with default values will be generated automatically next time the program is started. The file format is illustrated below:

```
# Fourpot ver.  1.20 parameter file

   36   32   20    0    0
    1    0    2    1    1    0    0    0   0
  350   500  0.90  0.90  0.80  0.00   0.0  0.0
  75.0   10.0  52000.0  0.500  0.01   0.0  0.0

2-D Fourier transform analysis
Original data
Interpolated data
Padded data
Fourier transform
Inverse data
Difference
Radial ave. spectre
X
Y
Field
Kx
Ky
Log_10$(F)
Kr
Log(F)
Log(F/C)
Magnetic data
```

```
Gravity data
Undefined data
Dimension:
Spacing:
Nyquist:
Wave length:
Rad. spectre depths:
Tensor rotation:
Rotation:
Declin. rotation:
Elev & Azimuth:
Distance
Depth

Low-Pass filtered:
High-Pass filtered:
Direction filtered:
Upward continued
Downward continued
Reduced to pole
dF/dz gradient
dF/dx gradient
dF/dy gradient
d^2$F/dz^2$ gradient
d^2$F/dx^2$ gradient
d^2$F/dy^2$ gradient
d^2$F/dxdy gradient
d^2$F/dydz gradient
d^2$F/dydz gradient
Horizontal gradient
Total gradient
Tilt gradient
Pseudo gravity
Pseudo magnetic
Gravity potential
Magnetic potential
Theta map
Rot. invariant 1
Rot. invariant 2
Reduced to equator
(d^2$F/dy^2$-d^2$F/dx^2$)/2
Max. horizontal gradient
Hilbert transform
Sun shading
General derivative
Unprocessed data
```

- The first line is used as a header to identify the file and version number.
- The 1.st relevant line defines three character height values: 1) main title, 2) axes labels and 3) the plot information text. The other two values are reserved for future use.
- The 2.nd line defines some (integer valued) options: 1) the screen mode (0/1) between the normal 4:3 aspect ratio and wide screen mode, 2) the color scale (0-6), 3) padding mode (0-3), 4) tapering shift modes (0/1), 5) contour vs. image map (0/1), 6) automatic low-pass

filtering (0/1) and 7) step number vs. step distance (0/1). The extra parameters are reserved for future use.

- The 3.rd line defines the *x*- (horizontal) and *y*- (vertical) position of the origin of the main graph (in pixels) from the bottom-left corner of the page (integers), and the length of the *x*- and *y*-axis relative to the size of the (origin subtracted) width and height of the plot area. The total size of the plot area is always 2970×2100 pixels (landscape A4). The fifth parameter defines the aspect ratio of the graph area for widescreen mode. Aspect ratio 1 refers to normal 4:3 screen and values less to zero indicate widescreen displays. The remaining parameters are reserved for future use.
- The 4.th line defines the initial values for the magnetic and gravity field components used in pole reduction and pseudo-field computations. These parameters include: 1) the inclination and 2) declination (in degrees from horizontal plane and true north) and 3) intensity (in nano-Teslas) of the magnetic field, 4) the density contrast (in grams per cubic centimeter) and 5) susceptibility (dimensionless in SI units). The extra parameters are reserved for future use.
- The following block of text lines define various text labels used in the graphs. The maximum length of each text label is 70 characters.
- The last block of text lines define the labels of the frequency domain operations shown in the graph (max. 70 characters).

The format of the *.DIS file is likely to change in the future. Also note that the ^ character (caret) is used to define superscripts (exponents), the _ character (underscore) is used to generate subscripts, and the $ character (dollar) is used to move the text back to the baseline (for example, `Log_10$(F)`).

6. Additional information

FOURPOT got started in 2003 when I worked at the Geological Survey of Finland for the 3-D crustal model project funded by the Academy of Finland. The original idea was to utilize the depth analysis methods of Ruotoistenmäki (1987). After I become lecturer of applied geophysics at the University of Oulu in 2005, I started to modify FOURPOT for educational purposes. Since then I've used it teaching gravity and magnetic data processing. Every now and then I've added new features and fixed bugs to make it more and more useful.

The Fourier transform is based on the FFT algorithm FORK by Jon Claerbout (1976). The bi-linear interpolation algorithm is adapted from Numerical Recipes (Press et al., 1988). Most of the Fourier operations have been documented in Blakely (1995) and Cooper and Cowan (2011). The principal directions are computed using RIDGEORIENT algorithm for fingerprint detection by Raymond Thai & Peter Kovesi (http://www.csse.uwa.edu.au/~pk/research/matlabfns/FingerPrints/ridgeorient.m). Thanks to Richard Stuart for his comments on smooth frequency filtering.

FOURPOT is written in Fortran 90 and compiled with Intel Visual Fortran 11.1. The graphical user interface is based on the DISLIN graphics library version 10.2 (http://www.dislin.de) by Helmut Michels. In principle, FOURPOT could be compiled and run on other operating systems (Linux, Mac OSX) without major modifications provided that DISLIN is also installed. At the moment, however, the source code is not made available and I do not provide active support for the software. Nonetheless, if you find the results erroneous or if you have suggestions for improvements, please, inform me.

7. References

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Blakely, R.J., 1995. Potential theory in gravity and magnetic applications. Cambridge Univ. Press, 441 p.

Claerbout, J., 1976. Fundamentals of geophysical data processing. With applications to petroleum prospecting. McGraw-Hill Book Co.

Cooper, G.R.J. & Cowan, D.R., 2006. Enhancing potential field data using fuilters based on the local phase. Computers & Geosciences , 1585-1591.

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Pedersen L.B. and Rasmussen T.M., 1990. The gradient tensor of potential field anomalies: some implications on data collection and data processing of maps: Geophysics, 55, 1558-1566.

Press, W.H., Flannery, B.P., Teukolsky, S.A., & Vetterling, W.T., 1988. Numerical recipes (in C) - The art of scientific computing. Cambridge Univ. Press, 735 p.

Ruotoistenmäki, T., 1987. Estimation of depth to potential field sources using the Fourier amplitude spectrum. Bulletin 340, Geological Survey of Finland. (PhD thesis).

8. Terms of use and disclaimer

The FOURPOT software is free for personal and academic use. It should not be used for commercial purposes without permission and must not be distributed onwards for profit. If you decide to publish results computed with FOURPOT, please, make a reference to this user manual and FOURPOT homepage at https://wiki.oulu.fi/display/~mpi@oulu.fi/2D+Fourier+domain+operations. If you find FOURPOT useful, please, send me a postcard ☺.

The FOURPOT software is provided as is. The author (MP) and the University of Oulu disclaim all warranties, expressed or implied, with regard to this software. In no event shall the author or the University of Oulu be liable for any indirect or consequential damages or any damages whatsoever resulting from loss of use, data or profits, arising out of or in connection with the use or performance of this software.

9. Contact information

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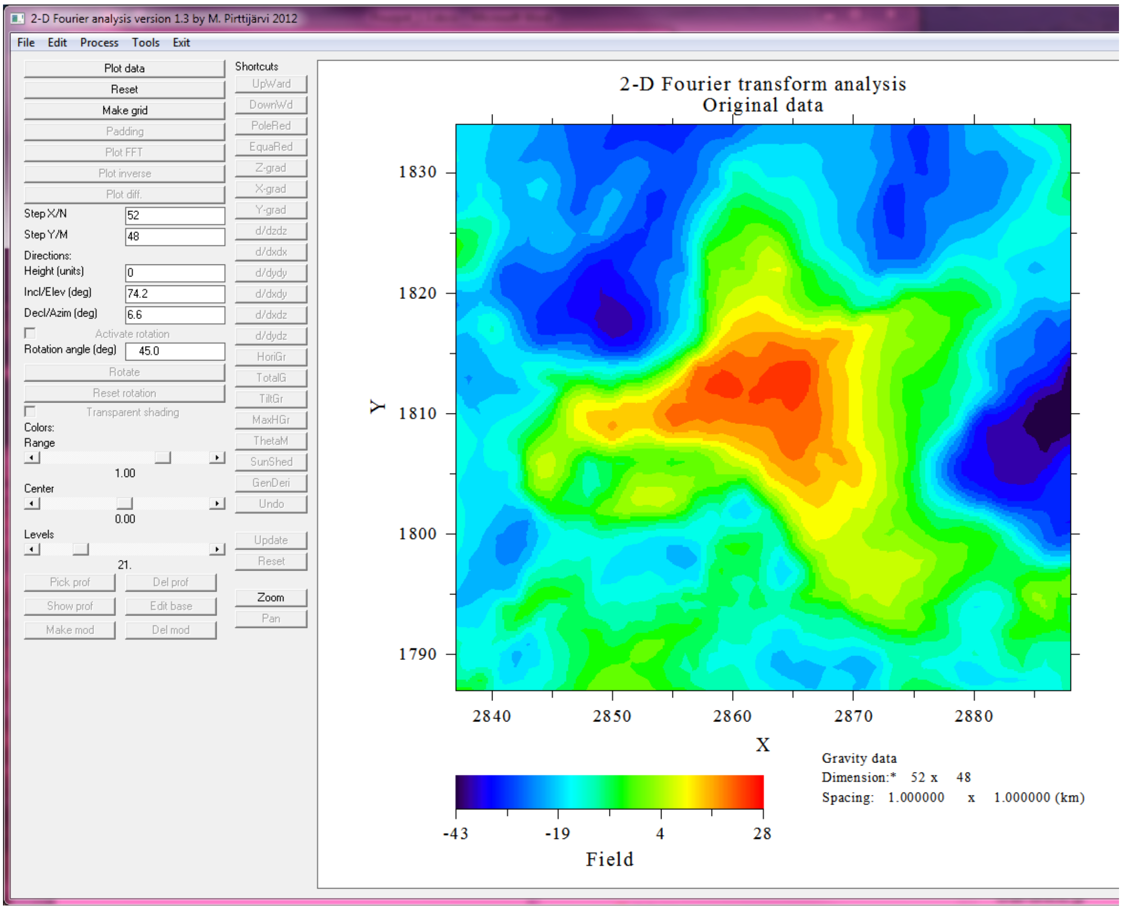
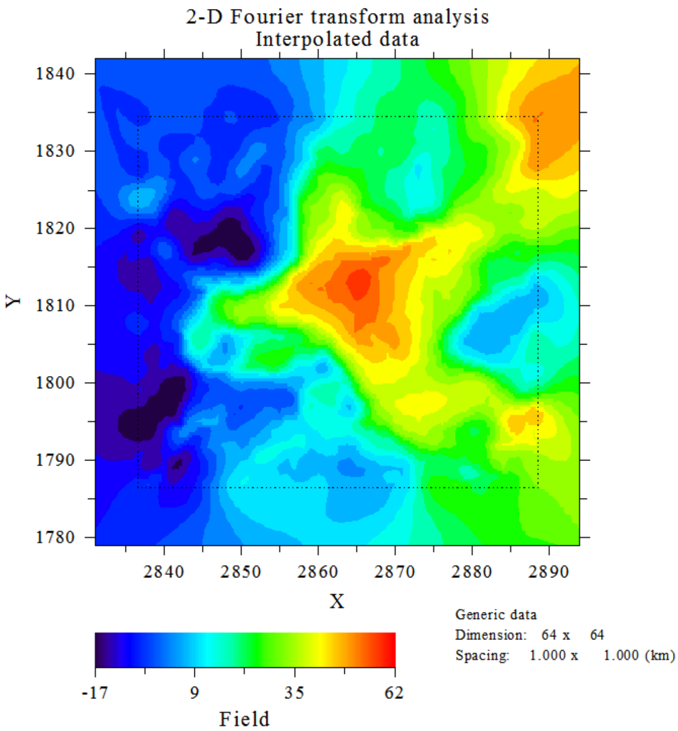
URL: http://www.gf.oulu.fi/~mpi

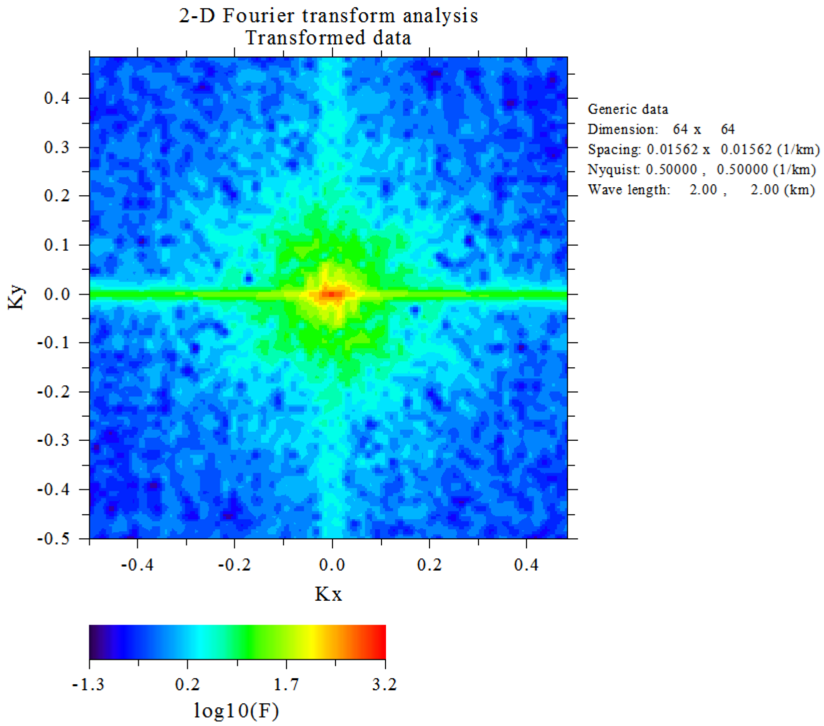
Appendix A. FOURPOT GUI after data input and default interpolation



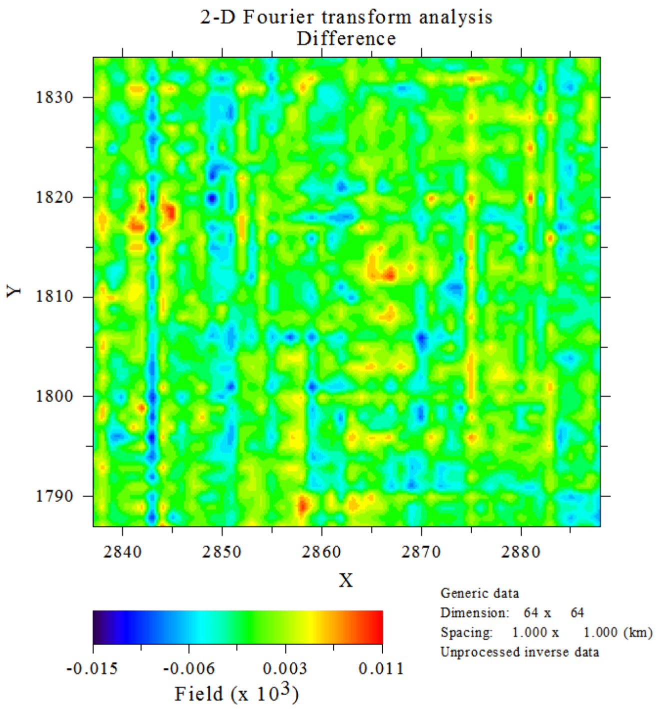
Appendix B. Data map after padding (with shift) and tapering



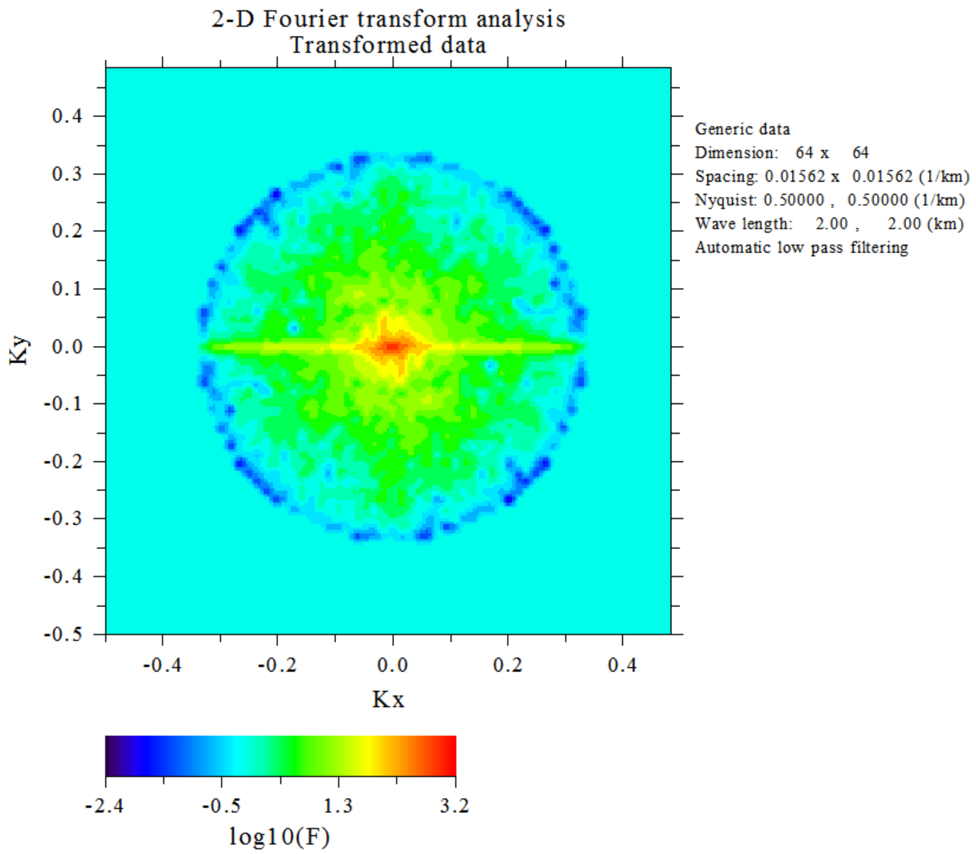
Appendix C. 2-D frequency (amplitude) spectrum



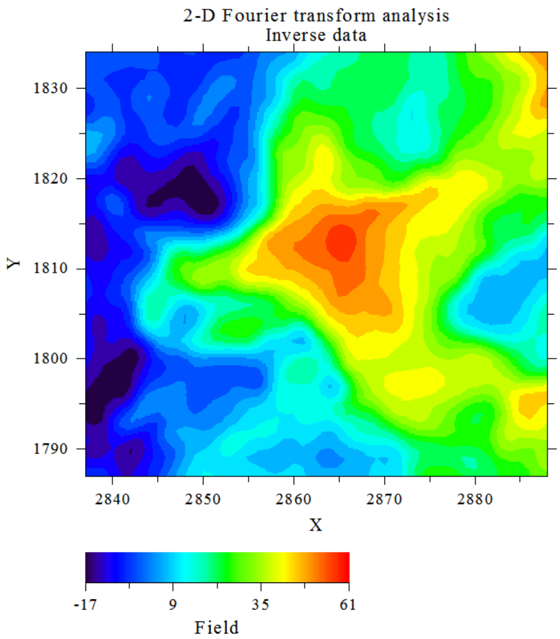
Appendix D. Difference between original data and inverse transformed data

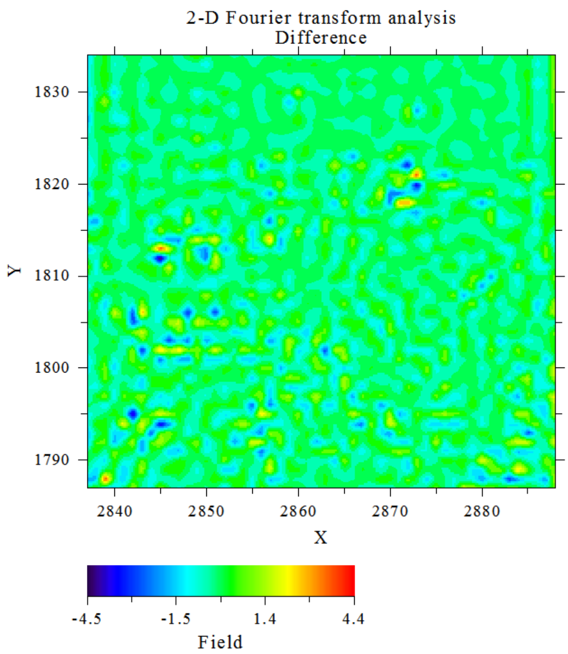


Appendix E. Low-pass filtered frequency spectrum

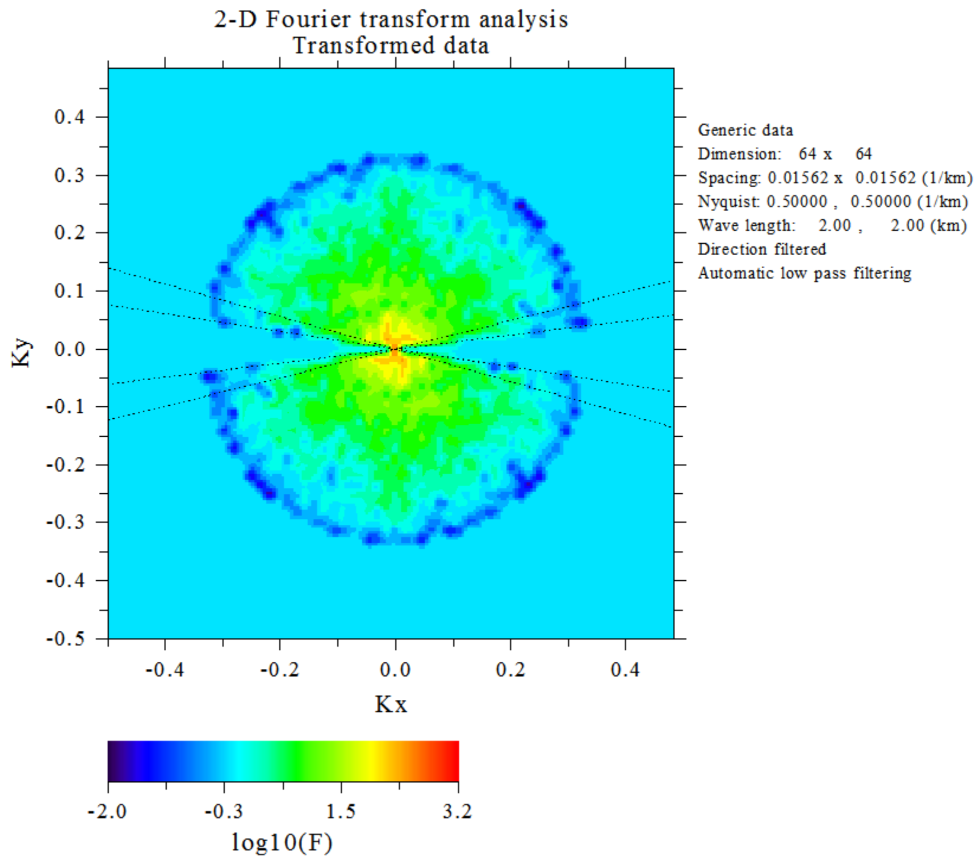


Appendix F. Low-pass filtered data and difference with the original data

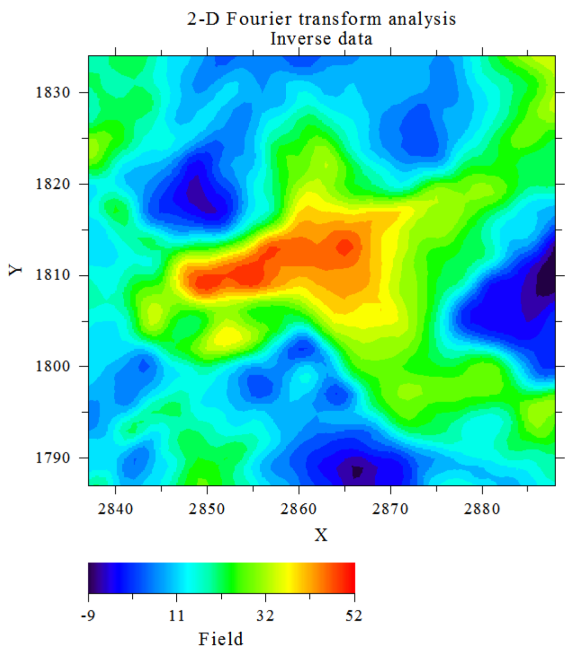


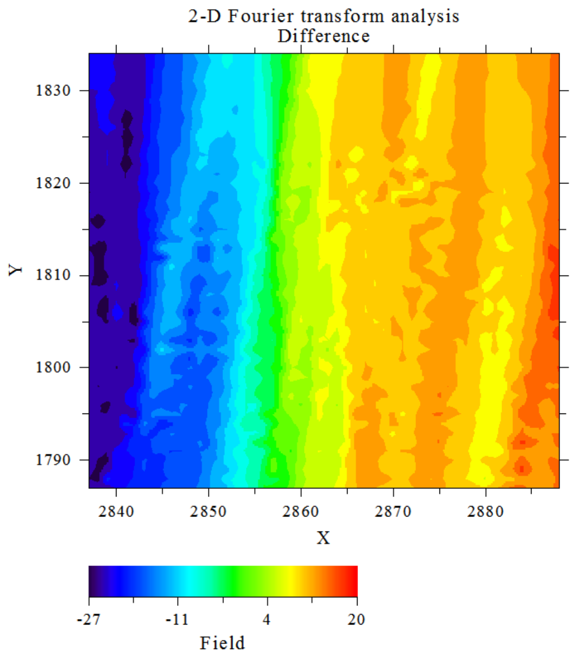


Appendix G. Direction filtered frequency spectrum

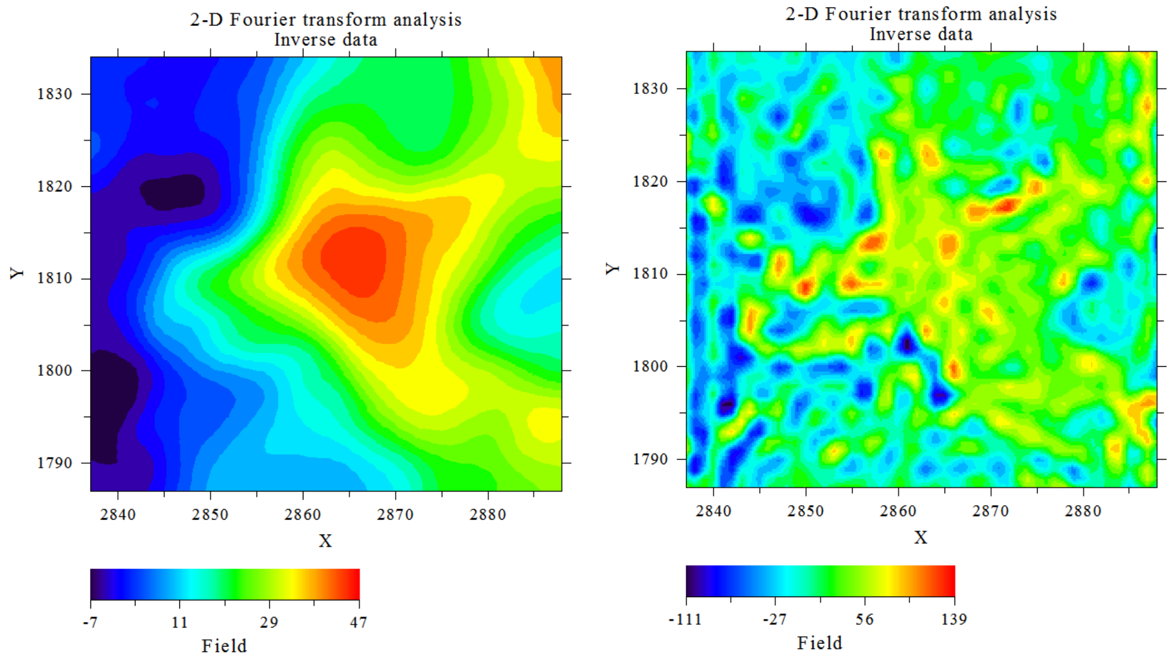


Appendix H. Direction filtered data and difference with the original data

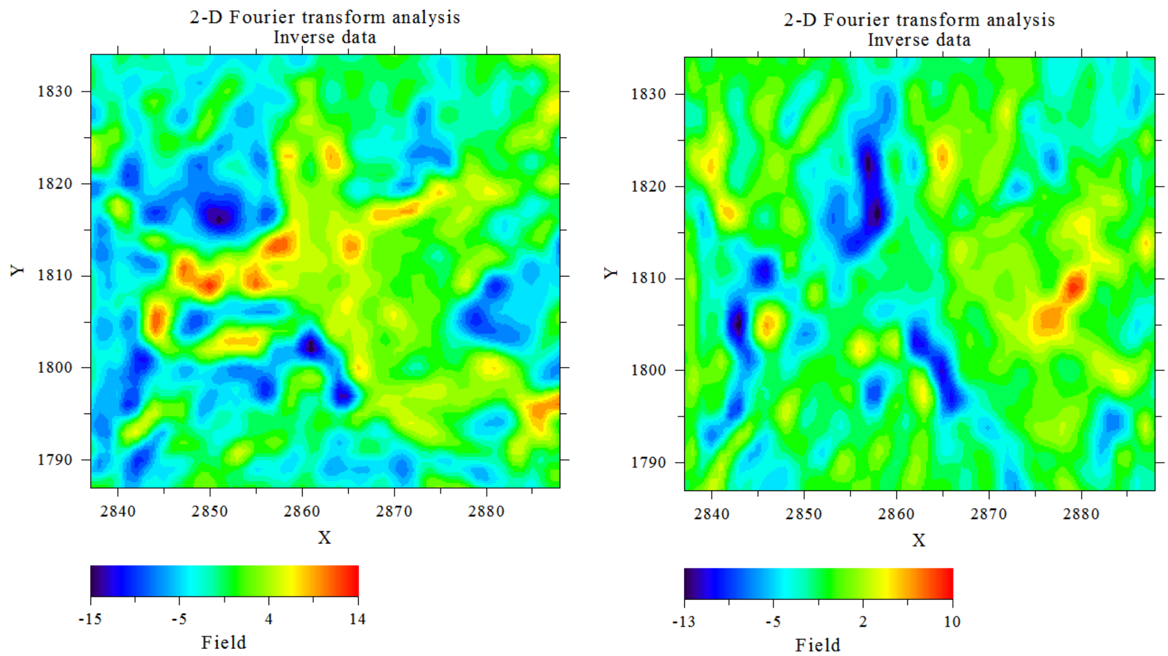




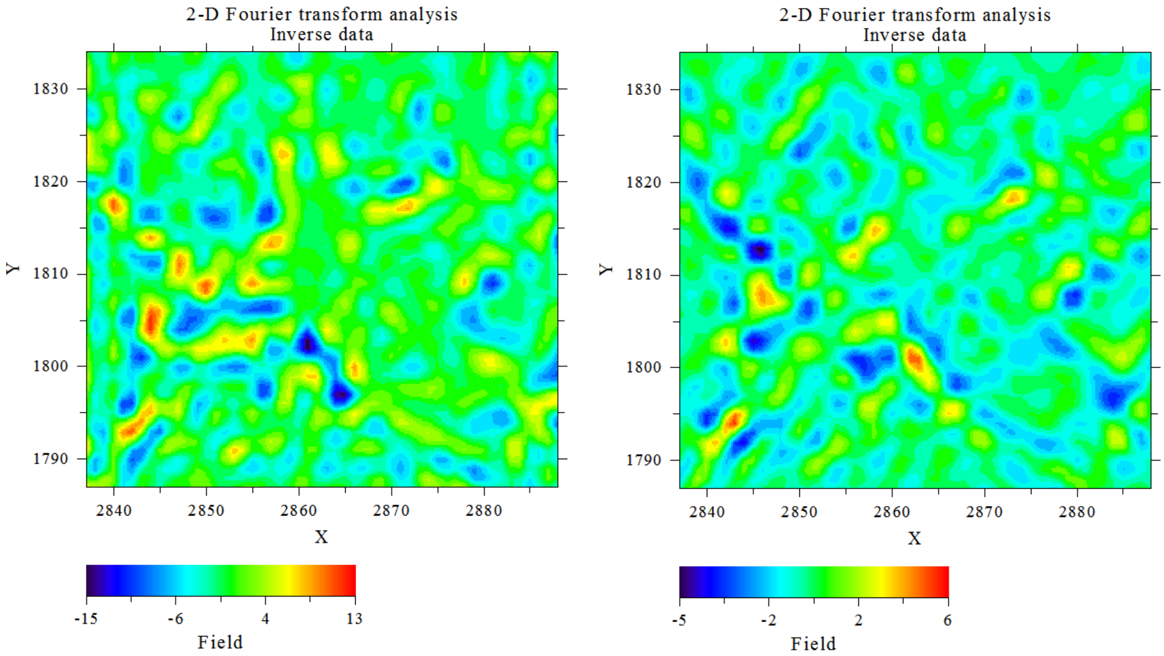
Appendix I. Upward and downward continued data (h= 2 km)



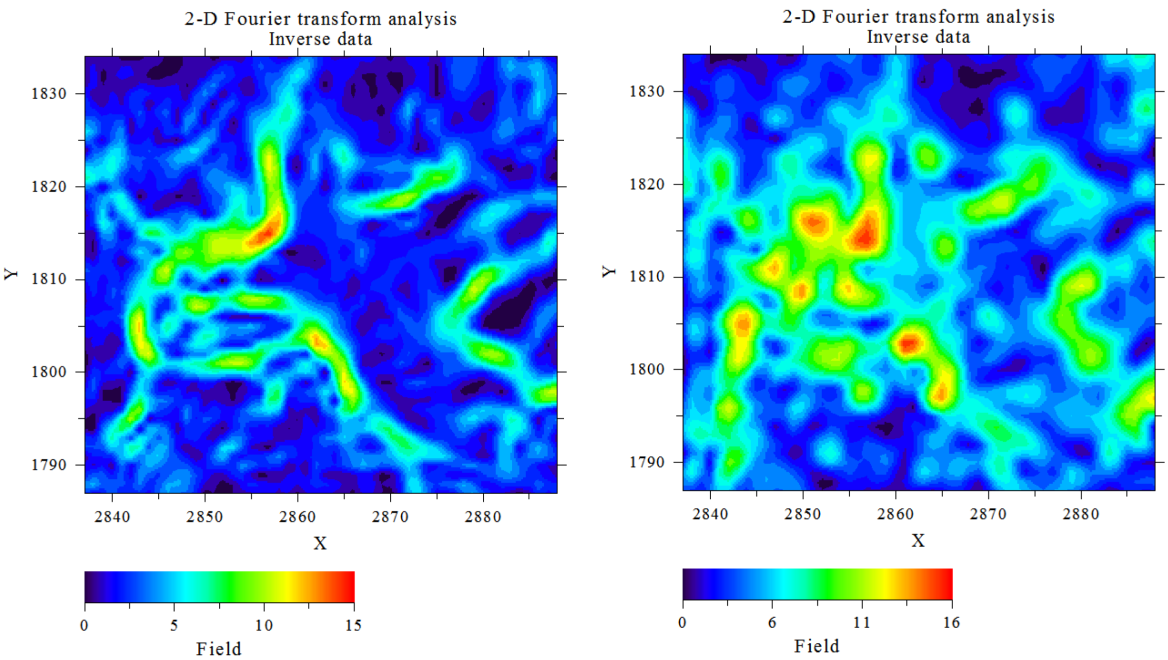
Appendix J. First vertical and horizontal x gradient



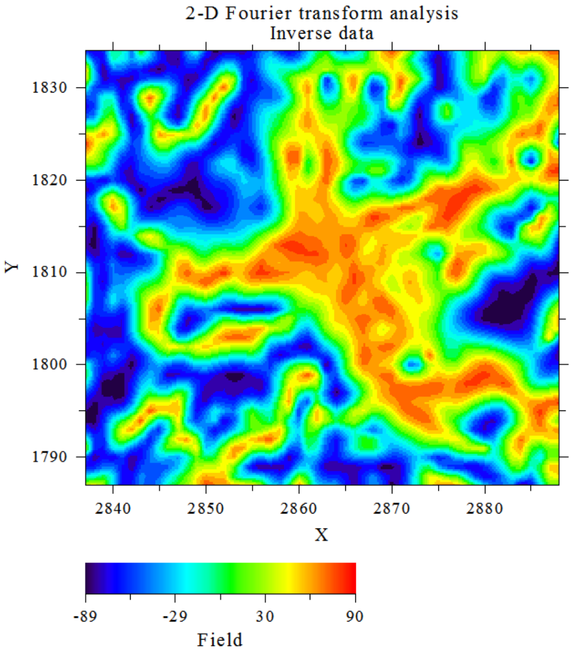
Appendix K. Second vertical and horizontal xy gradient

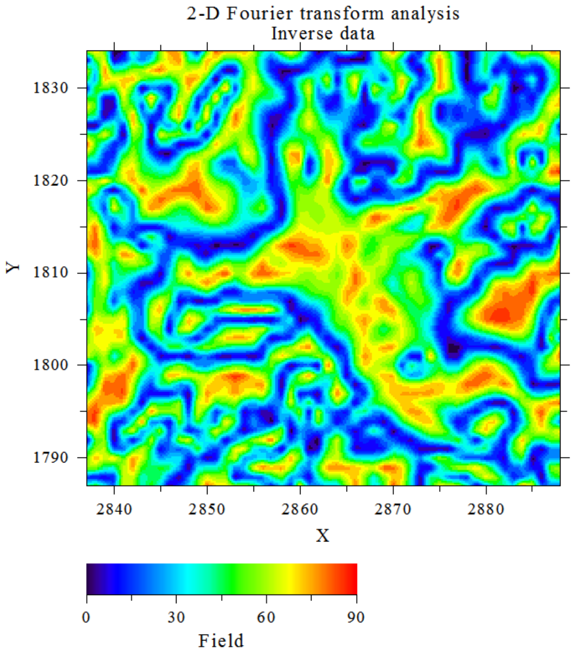


Appendix L. Horizontal gradient and total gradient (analytical signal)

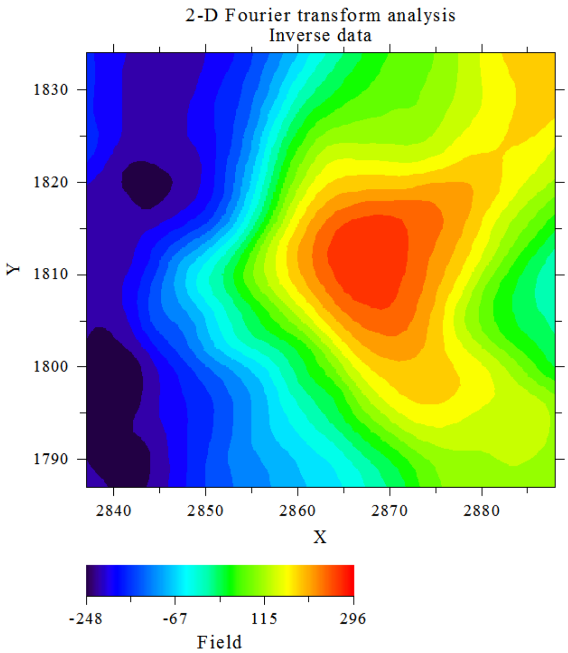


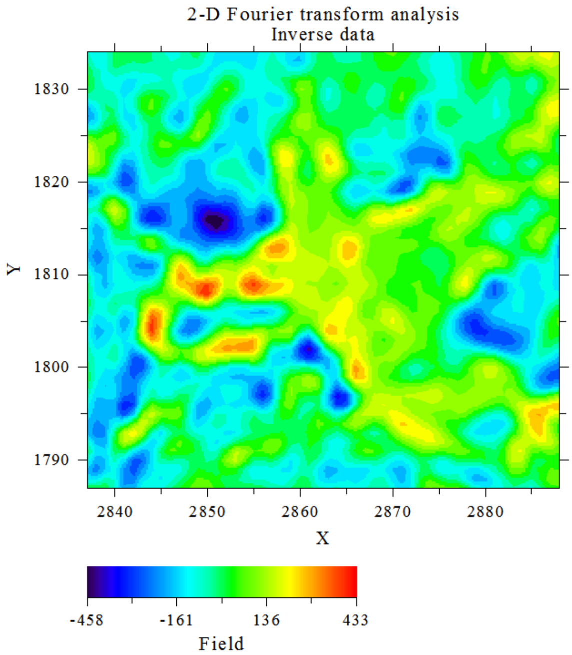
Appendix M. Tilt derivative and theta map



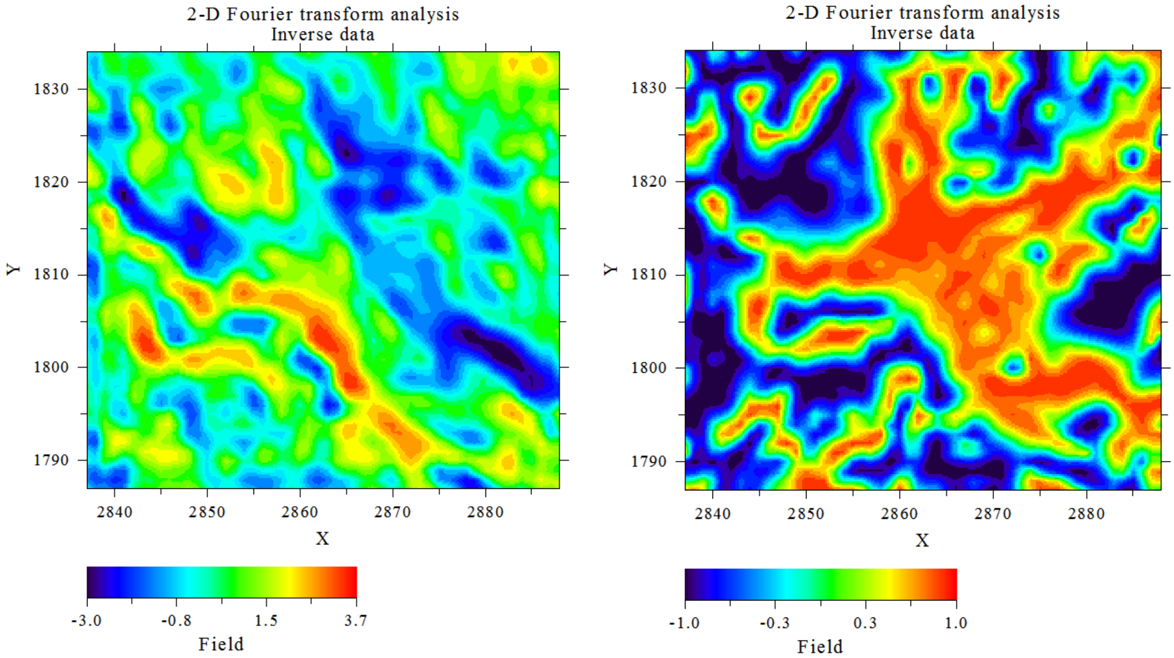


Appendix N. Gravity potential and pseudo-magnetic field (k= 0,01 SI, $\Delta\rho$= 0,2 g/cm^3, and T_0= 52000 nT)

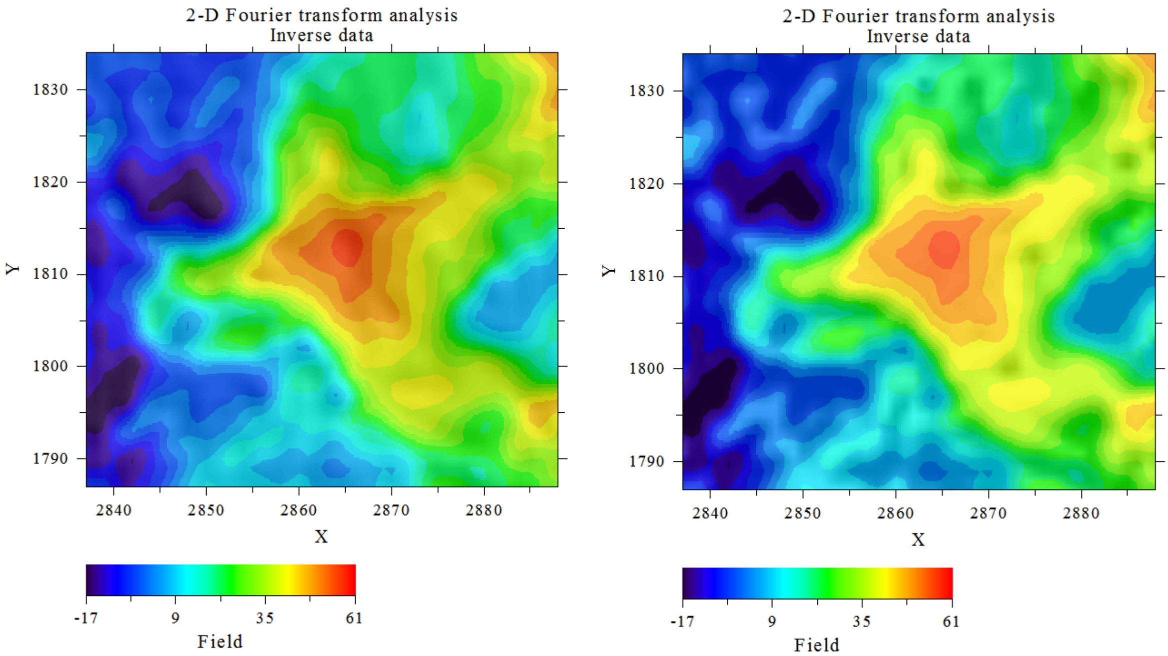




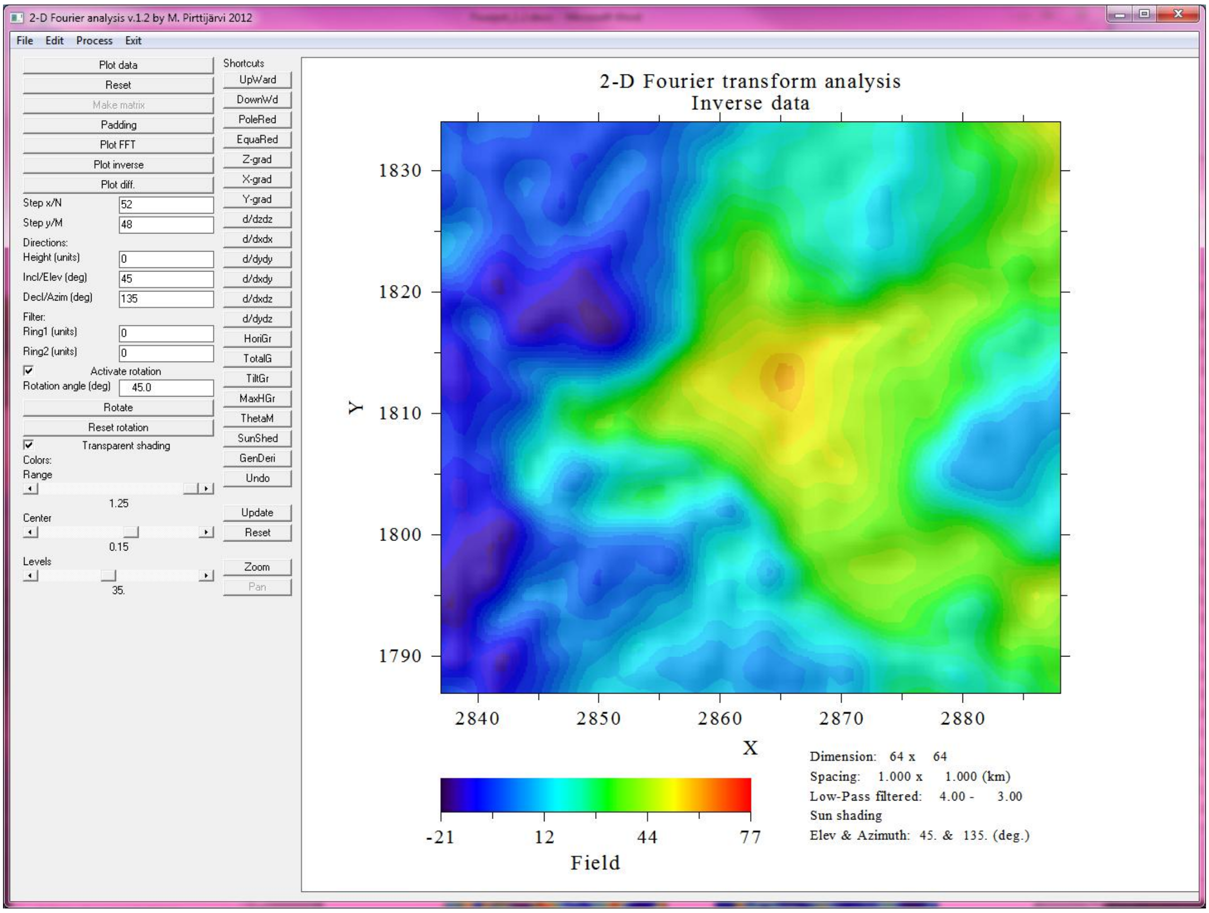
Appendix O. Sun shading and general derivative (Elev. 75 deg. and Azim 10 deg.)



Appendix P. Sun shading and general derivative (Elev. 75 deg. and Azim 10 deg.) shown transparently on the interpolated (low-pass filtered) data



Appendix Q. FOURPOT GUI + Sun shading (Elev. 20 deg. and Azim 145 deg.) shown transparently on the low-pass filtered data using slightly altered color scale



Appendix R. Ridge picking (3x3) made on max. horizontal gradient and principal directions shown on top of horizontal gradient